



# **NAVAL POSTGRADUATE SCHOOL**

**MONTEREY, CALIFORNIA**

## **THESIS**

**GEOLOCATION OF WIMAX SUBSCRIBERS STATIONS  
BASED ON THE TIMING ADJUST RANGING  
PARAMETER**

by

Don E. Barber Jr.

December 2009

Thesis Advisor:  
Co-Advisor:  
Second Reader:

John McEachen  
Herschel Loomis  
Vicente Garcia

**Approved for public release; distribution is unlimited**

<b>REPORT DOCUMENTATION PAGE</b>			<i>Form Approved OMB No. 0704-0188</i>	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington DC 20503.				
<b>1. AGENCY USE ONLY (Leave blank)</b>		<b>2. REPORT DATE</b> December 2009	<b>3. REPORT TYPE AND DATES COVERED</b> Master's Thesis	
<b>4. TITLE AND SUBTITLE</b> Geolocation of WiMAX Subscriber Stations Based on the Timing Adjust Ranging Parameter			<b>5. FUNDING NUMBERS</b>	
<b>6. AUTHOR</b> Don E. Barber Jr.				
<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b> Naval Postgraduate School Monterey, CA 93943-5000			<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>	
<b>9. SPONSORING /MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b> N/A			<b>10. SPONSORING/MONITORING AGENCY REPORT NUMBER</b>	
<b>11. SUPPLEMENTARY NOTES</b> The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.				
<b>12a. DISTRIBUTION / AVAILABILITY STATEMENT</b> Approved for public release; distribution is unlimited			<b>12b. DISTRIBUTION CODE</b>	
<b>13. ABSTRACT</b>  This thesis investigates the possibility of geolocating a WiMAX subscriber station based on the timing adjust ranging parameter within the network signal internals. The basic approach to geolocation based on radial distances from multiple base stations is outlined. Specifics of the timing parameters used during WiMAX network entry are examined as they relate to calculating these distances. Laboratory testing demonstrates successful capture of ranging parameters from the air interface, leading to the development of a web based geolocation tool to map likely locations of subscriber stations. Field collection of the air interface from a single base station network verified a high correlation with low variance when comparing values in timing adjust values in packets exchanged during network entry. Using field test results, computer simulation further refined the expected geolocation accuracy in multiple base-station networks. Results show the possibility of fixes with 10 times greater accuracy than in previous results in literature applying timing advance techniques to Global System for Mobile communications networks.				
<b>14. SUBJECT TERMS</b>  802.16, WiMAX, Geolocation, Ranging, Timing Adjust			<b>15. NUMBER OF PAGES</b> 95	
			<b>16. PRICE CODE</b>	
<b>17. SECURITY CLASSIFICATION OF REPORT</b> Unclassified	<b>18. SECURITY CLASSIFICATION OF THIS PAGE</b> Unclassified	<b>19. SECURITY CLASSIFICATION OF ABSTRACT</b> Unclassified	<b>20. LIMITATION OF ABSTRACT</b> UU	

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)  
Prescribed by ANSI Std. Z39-18

THIS PAGE INTENTIONALLY LEFT BLANK

**Approved for public release; distribution is unlimited**

**GEOLOCATION OF WIMAX SUBSCRIBER STATIONS  
BASED ON THE TIMING ADJUST RANGING PARAMETER**

Don E. Barber Jr.  
Lieutenant, United States Navy  
B.S., United States Naval Academy, 2004

Submitted in partial fulfillment of the  
requirements for the degree of

**MASTER OF SCIENCE IN ELECTRICAL ENGINEERING**

from the

**NAVAL POSTGRADUATE SCHOOL  
December 2009**

Author: Don E. Barber

Approved by: John C. McEachen  
Thesis Advisor

Herschel H. Loomis  
Co-Advisor

Vicente C. Garcia  
Second Reader

Jeffrey B. Knorr  
Chairman, Department of Electrical and Computer Engineering

THIS PAGE INTENTIONALLY LEFT BLANK

## **ABSTRACT**

This thesis investigates the possibility of geolocating a WiMAX subscriber station based on the timing adjust ranging parameter within the network signal internals. The basic approach to geolocation based on radial distances from multiple base stations is outlined. Specifics of the timing parameters used during WiMAX network entry are examined as they relate to calculating these distances. Laboratory testing demonstrates successful capture of ranging parameters from the air interface, leading to the development of a web based geolocation tool to map likely locations of subscriber stations. Field collection of the air interface from a single base station network verified a high correlation with low variance when comparing values in timing adjust values in packets exchanged during network entry. Using field test results, computer simulation further refined the expected geolocation accuracy in multiple base-station networks. Results show the possibility of fixes with 10 times greater accuracy than in previous results in literature applying timing advance techniques to Global System for Mobile communications networks.

THIS PAGE INTENTIONALLY LEFT BLANK

# TABLE OF CONTENTS

<b>I.</b>	<b>INTRODUCTION.....</b>	<b>1</b>
A.	BACKGROUND: WIMAX AND WHY WE CARE .....	1
B.	OBJECTIVE: GEOLOCATION .....	3
C.	RELATED WORK: METHODS OF GEOLOCATION .....	3
D.	APPROACH.....	5
E.	THESIS ORGANIZATION.....	6
<b>II.</b>	<b>WIMAX WORKINGS.....</b>	<b>7</b>
A.	NETWORK ENTRY AND RANGING .....	8
B.	RANGE RESPONSE MESSAGE .....	11
C.	TIMING ADJUST .....	12
<b>III.</b>	<b>LABORATORY OBSERVATIONS .....</b>	<b>13</b>
A.	INITIAL OBSERVATIONS IN THE NPS NETWORKS LAB.....	13
B.	OBSERVATIONS OF TEST DATA FROM SANJOLE.....	16
<b>IV.</b>	<b>INTERFACE DEVELOPMENT.....</b>	<b>19</b>
A.	GUI BACKGROUND.....	19
B.	ENABLING APPROXIMATIONS.....	20
C.	LIKELY LOCATION CALCULATIONS .....	21
D.	DISPLAYED RESULTS .....	25
<b>V.</b>	<b>FIELD EXPERIMENTS AND RESULTS .....</b>	<b>27</b>
A.	TESTING LOCATION, CONFIGURATION, AND PROCEDURE .....	27
B.	NOTED CHALLENGES DURING FIELD TESTS.....	29
C.	FIELD TEST RESULTS.....	31
<b>VI.</b>	<b>MULTIPLE BASE STATION SIMULATIONS .....</b>	<b>35</b>
A.	TWO BASE STATION SIMULATION .....	35
B.	MULTIPLE BASE STATION SIMULATIONS .....	36
<b>VII.</b>	<b>CONCLUSIONS AND RECOMMENDATIONS.....</b>	<b>43</b>
A.	CONCLUSIONS .....	43
B.	RECOMMENDATIONS.....	44
<b>APPENDIX I.</b>	<b>WIMAX CERTIFICATION PROFILES.....</b>	<b>45</b>
<b>APPENDIX II.</b>	<b>802.16D-2004 OFDM SYMBOL PARAMETERS.....</b>	<b>47</b>
<b>APPENDIX III.</b>	<b>GUI HTML/JAVASCRIPT CODE.....</b>	<b>49</b>
<b>APPENDIX IV.</b>	<b>FIELD TEST IMAGES.....</b>	<b>57</b>
<b>APPENDIX V.</b>	<b>MULTIPLE BASE STATION SIMULATIONS .....</b>	<b>61</b>
A.	TWO BASE STATIONS THROUGH VARYING ANGLES.....	61
1.	Two Base Stations Through Varying Angle MATLAB Code.....	61
2.	Two Base Stations through Varying Angles Example Plots .....	63
B.	MULTIPLE BASE STATIONS.....	64



1.	Multiple Base Stations MATLAB Code.....	64
2.	Random Angle and Distance Example Plots .....	67
3.	Evenly Spaced Angles with Random Distance Example Plots.....	68
4.	Evenly Spaced Angles with Fixed Radial Distance Example Plots .....	69
LIST OF REFERENCES .....		71
INITIAL DISTRIBUTION LIST .....		73

## LIST OF FIGURES

Figure 1.	WiMAX Forum WiMAX Deployment Map .	2
Figure 2.	Illustration of Overlapping Timing Advance Range Rings.	5
Figure 3.	WiMAX Frame Format.....	9
Figure 4.	Ranging Procedure .....	10
Figure 5.	Network Entry Process .	11
Figure 6.	Laboratory RNG-RSP TA vs Distance.	15
Figure 7.	Flow Chart for Geolocation Method.....	22
Figure 8.	Illustration of Geometry to Calculate Circle Intersections.	23
Figure 9.	Sample GUI Screen Shot.	26
Figure 10.	Test Configuration.	28
Figure 11.	Field Collection Station Locations.....	29
Figure 12.	Field Test RNG-RSP TA vs Distance.....	31
Figure 13.	Cummulative TA Probability Distribution from all Ranges.....	33
Figure 14.	Distance to Midpoint Between Intersections with 2 BS Varying Angle.	36
Figure 15.	Inaccurate Fix Situation with 3 BS.	37
Figure 16.	Average Distance from Estimate to SS with Multiple BS.....	38
Figure 17.	Comparision of Simulations with Different Distance per TA.	40
Figure 18.	Circular Error Probable from Multiple BS Simulations.	40
Figure 19.	Base Station.	57
Figure 20.	Subscriber Station.	57
Figure 21.	Collection Vehicle with Roof Mounted Antenna.	58
Figure 22.	Collection Suite (GPS, Laptop, and WaveJudge).....	58
Figure 23.	Screen Shot of WaveJudge Interface with Captured RNG-RSP Open.....	59
Figure 24.	Sample Plots from 2 BS Simulation with Varying Angles and Distances.	63
Figure 25.	Sample Multilple BS Plots with Random Angles and Distances.	67
Figure 26.	Sample Multilple BS Plots with Equal Angles and Random Distances.	68
Figure 27.	Sample Multilple BS Plots with Equal Angles and Fixed 1 km Distances.	69

THIS PAGE INTENTIONALLY LEFT BLANK

## LIST OF TABLES

Table 1.	TA Definitions from 802.16 Standards .....	11
Table 2.	Sample RNG-RSP Values. ....	14
Table 3.	Field Collection Results.....	32
Table 4.	Fixed and Mobile WiMAX Certification Profiles .....	45
Table 5.	OFDM Symbol Parameters .....	47

THIS PAGE INTENTIONALLY LEFT BLANK

## **LIST OF ACRONYMS AND ABBREVIATIONS**

AOA	Angle of Arrival
BS	Base Station
BPSK	Binary Phase Shift Keying
CEP	Circular Error Probable
CDMA	Code Division Multiple Access
CID	Connection Identifier
FCH	Frame Control Header
FDOA	Frequency Difference of Arrival
GPS	Global Positioning System
GSM	Global System for Mobile Communications
GUI	Graphic User Interface
IP	Internet Protocol
ISI	Inter-Symbol Interference
IFFT	Inverse Fast Fourier Transform
LTE	Long Term Evolution
MAC	Media Access Control
OFDM	Orthogonal Frequency Division Multiplexing
PDF	Probability Density Function
PN	Pseudo Noise
RF	Radio Frequency
RNG-REQ	Ranging Request
RNG-RSP	Ranging Response
RSSI	Received Signal Strength Indication
SS	Subscriber Station
TDOA	Time Difference of Arrival
TDD	Time Division Duplexing
TDMA	Time Division Multiple Access
TA	Timing Adjust
USB	Universal Serial Bus
VoIP	Voice over Internet Protocol
WiMAX	Worldwide Interoperability for Microwave Access

THIS PAGE INTENTIONALLY LEFT BLANK

## **EXECUTIVE SUMMARY**

WiMAX is an important emergent technology. It can provide fixed data access in developing areas without the costly need for cable infrastructure and is poised as one of the two technologies that will replace current 3G cellular network as infrastructure converges on a voice over internet protocol network and mobile subscribers use more and more data. Since WiMAX's high speed connection is wireless, a fixed or mobile subscriber's location is not predetermined. There are many instances in which locating a WiMAX subscriber may be important including emergency response to a medical emergency or fire and aiding in law enforcement and homeland security.

There are many methods to geolocate a radio frequency device, and each has tradeoffs and limitations. Time difference of arrival requires very precisely synchronized receivers and frequency difference of arrival requires significant velocities to generate Doppler shifts. While one solution is to simply install GPS chips in subscriber units, this adds cost to manufacture, and requires the subscriber unit to be cooperative to provide its location. Rather than relying on a phone to transmit its own internally collected location, a network-ranging feature built into the WiMAX standard called timing adjust can be exploited to derive the distance from known points, such as cell towers, to establish a geolocation.

In a WiMAX network, similar to a Global System for Mobile communications cellular network, different stations take turns transmitting at different times to share the link. However, the messages of subscribers at different distances from the tower take different lengths of time to reach the tower depending on their distance to the tower. To correct for this so that the base station receives each subscriber's data in the appropriate time slot without interfering with other users, the base station sends out timing adjusts to tell each individual subscriber to transmit sooner or later so its data arrives in the right time window.

By listening to this exchange over the air interface, it is possible to extrapolate what distance away from the tower a subscriber is, based on the propagation speed, the speed of light, and how many timing adjust units the base station tells the subscriber to



use. Previous studies had explored using this method to geolocate Global System for Mobile communications cell phones, and the objective of this thesis was to apply these principles to WiMAX networks despite different radio frequency parameters and message formats.

WiMAX was developed to support a common media access layer on top of different physical layers (different frequencies and bandwidths), so the signaling parameters may vary from network to network and in different regions of the world, but overall principles will consistently apply across standard compliant WiMAX devices. For testing, a network was used that resulted in a theoretical distance per unit of timing adjust of 52 m based on the physical layer parameters.

After initial laboratory testing confirmed that the appropriate timing-adjust messages could be captured from the air interface and conformed to the format expected from the standard's documentation, a methodology for geolocation based on two collected radii was developed and implemented in a web-based mapping interface. Taking a collection suite consisting of a laptop, GPS unit, and one collection and analysis box, vehicle-based field collection was then conducted at distances more realistic to a deployed cellular network simulating real world application.

Testing in the laboratory and field tests both showed that timing adjust did in fact linearly correlate to distance, making it possible to approximate the subscriber unit's distance from the base station based on information collected from the air interface. Further, repeated captures of ranging packets at fixed distances showed very low variance in the timing adjust value received, again indicating the ability to accurately geolocate a subscriber based on this information.

Though limited to one base station for testing, computer simulation using the results of the tests from the single base station extended the results to model numerous multiple base-station networks. Results from the simulation showed that in networks with various numbers of towers and random tower placements, the location of a subscriber could generally be determined within 50 meters if accurate tower locations and timing adjust offsets are known.

Geolocation within 50 meters based on passive collection of the air interface offers great potential, both to cellular networks hoping to offer location based services and to emergency response and tactical personnel who may need to locate mobile persons of interest. Both testing and simulation demonstrate the possibility of developing such a fieldable capability in a WiMAX network based on signal internals captured from the air interface.

THIS PAGE INTENTIONALLY LEFT BLANK

## **ACKNOWLEDGMENTS**

At the outset, I would like to take a moment to acknowledge some of the many people who helped make this thesis possible for me. First, my indebted thanks to my advisors, Dr. McEachen and Dr. Loomis, for their insights and guidance throughout the thesis process. Special thanks to Prof. Bob Broadston for facilitating use of the WiMAX and analysis equipment both on and off site, to Marianna Verett for coordinating space and frequency availability for field testing, and to Dr. Les Carr for taking the time to review the math with me. Beyond WiMAX, I also extend my gratitude to Dr. Alex Julian and the Naval Postgraduate School's electromagnetic launch program for introducing me to graduate research, and Joe Noble for first introducing me to geolocation and empowering me as a technical manager as a young Lieutenant Junior Grade. Finally, to the friends and fellow officers who helped me get through the master's program and other trials at the postgraduate school, thank you for the support.

THIS PAGE INTENTIONALLY LEFT BLANK

## **I. INTRODUCTION**

### **A. BACKGROUND: WIMAX AND WHY WE CARE**

The IEEE 802.16-2001 standard published in April 2002 defines a wireless metropolitan area network, designed to distribute a high-speed networking capability without the costly need for cabled infrastructure. The standard was designed to evolve with a common media access control (MAC) protocol with different physical layer considerations dependent on the spectrum employed [1]. Since the initial publication in 2002, several related projects in the 802.16 working group have extended the scope and capabilities of the initial 2001 standard, most notably the 802.16d-2004 fixed standard and 802.16e-2005 mobile standard [2].

The fixed wireless standard provides the capability for leap-frog advancement in connectivity without the high costs associated with the need to run cable to every subscriber. In underdeveloped areas, including rural areas and developing nations, fixed 802.16d provides a means of distributing high-speed data transfer while completely bypassing the build out of a fiber or copper infrastructure. Even in urban areas, fixed 802.16d has been suggested as a backhaul technology for localized 802.11 hotspots.

The mobile standard has been envisioned as a fourth generation replacement to current cellular mobile subscriber phone and data services, increasing data throughput. Mobile 802.16e offers greater data rates than current 3G cellular technologies, while providing greater mobility than 802.11. Antennas for 802.16 can be added to current cell towers, taking advantage of the existing towers and backhaul links, while providing increased data throughput on the mobile network. The mobile 802.16 standard offers great opportunity for continuing convergence of voice and data networks and growing consumer demand for greater data throughput to mobile devices.

Just as Wi-Fi was developed as a common subset of the 802.11 standard to ensure compatibility between vendors, Worldwide Interoperability for Microwave Access (WiMAX) is an industry consortium to certify products that must commonly comply with

specified aspects of the 802.16 standard. While this leaves room for different hardware implementation and special features for vendors to differentiate themselves, it also ensures that there is commonality in initialization and establishing communications between base stations (BS) and subscriber stations (SS). Several PlugFest events have been hosted in coordination with the WiMAX forum where vendors demonstrated interoperability, and this interoperability is specifically advertised as a selling point [3],[4]. This commonality benefits industry through standardization of components that reduces cost through volume. It also benefits the consumer through competition between vendors and network administrators through the development of standardized diagnostic equipment to examine networks and links.

Already, many WiMAX networks have been built out globally, with many more planned. While economic recession has slowed investment in new technology and infrastructure, as noted above, in many cases using the leapfrog technology is still more cost effective. The map in Figure 1 shows currently deployed WiMAX systems across the globe [5].



Figure 1. WiMAX Forum WiMAX Deployment Map (From [5]).

Growing build out of an emerging global standard, with industrial partnerships to ensure interoperability, strongly suggests that WiMAX is the wave of the future, offering both mobility and high data throughput. As consumers, both civil and government, begin to use and rely on this new technology, emergent features and uses in the protocols are worthy of examination.

## **B. OBJECTIVE: GEOLOCATION**

With the potential for widespread deployment of WiMAX-compliant devices in the near future, one important consideration is how to locate subscribers in both fixed and mobile applications. Location-based services have grown increasingly popular in the current generation of cellular phones, providing weather, traffic, and navigation information and even social-networking services to identify friends in the area. Beyond commercial customer demand for location information, it is important to law enforcement for stolen property recovery and emergency response personnel who may need to be able to quickly locate people in emergency situations. Beginning in 2003, Congress mandated standards for mobile carriers to be able to provide accurate locations for the origins of 911 calls from mobile phones to respond to such situations [6].

Even beyond the scope of emergency response, there may be law enforcement or homeland security necessities to find the location of a threat using wireless or mobile communications. In light of these critical needs to locate subscribers of wireless technologies, and with WiMAX poised to be a dominant emerging wireless standard, this thesis's objective is to develop a method to geolocate WiMAX subscribers and assess the fix accuracy that can be achieved using this technique.

## **C. RELATED WORK: METHODS OF GEOLOCATION**

There are several means of addressing the issues of locating wireless devices through both handset-based and network-based solutions. A global position system (GPS) chip built into a mobile unit can provide accurate location data satisfying the Congressional requirement for an E-911 situation, but has several drawbacks. A GPS device increases the cost to manufacture and power draw on the mobile device, while



requiring the transmission of extra data. An external, passive approach to location trades off some location resolution but overcomes the cost, power, and bandwidth penalties of a GPS solution. It also continues to provide a location capability even if the GPS location capability should malfunction or be maliciously disabled, which may be critical in law enforcement or homeland security situations requiring location data.

Several possible passive external techniques exist to locate a radio frequency (RF) devices, including received-signal-strength indication (RSSI), angle of arrival (AOA), time difference of arrival (TDOA), frequency difference of arrival (FDOA), or potential internal signal characteristics [2],[7]. Direct application of RSSI has many limitations including multipath and variable broadcast strength and does not provide a robust, reliable means of location. FDOA requires significant relative motion to generate Doppler shift, and while platforms such as aircraft maybe able to employ it, it is infeasible for terrestrial geolocation. This leaves consideration of AOA, TDOA, and signal internals, or some combination thereof, as the best possibility for locating an 802.16 subscriber.

In looking to the future, it is best to begin with what has already been done. Currently, the nationwide time division multiple access (TDMA) and Global System for Mobile communications (GSM) cellular providers use network-based location technology; both Cingular and T-Mobile employ TDOA technologies. This approach is likely driven by the structure of the signal itself, since in both TDMA and GSM significant timing data is built in to control access to the shared spectrum. The code division multiple access (CDMA) providers, including Sprint and Verizon, have opted for an assisted GPS solution to meet the legal E-911 requirements imposed by Congress [6].

GSM mobile stations can be located within several hundred meters based on an internal parameter for time of arrival called timing advance. Timing advance is used by the network and handset to align the traffic bursts with the TDMA frames of GSM. Using this timing advance and speed of propagation, range rings from base station towers can be approximated, and the intersection of these range rings from multiple towers provides an approximate location for the mobile station [8]. Further refinement of this approach

found averaging multiple timing advance measurements minimized error in random variable sampling, tightening location accuracy [9].

#### **D. APPROACH**

The uplink in the 802.16 MAC is also shared between SSs in a TDMA fashion, with initial assignment of a timing adjust (TA) generated by the BS after the initial entry and ranging request by a SS [1]. This parallel to GSM creates an opportunity to leverage similar location techniques, dependent on access to the network at the BS or being able to receive this information over the air interface with the known BS locations. This is the approach to geolocation used in this thesis, using timing data in the signal internals to establish ranges to various known tower locations as has been previous explored with GSM.

To begin, the 802.16 standards were investigated to identify which packets contain this timing data, when they occur in traffic, and what the bits encoded in the packets mean. After establishing the expected parameters from specifications and literature, laboratory testing confirmed that these bits can be extracted from packets on the air interface in a controlled environment. Following laboratory testing, field testing confirmed linear correlation between TA and real-world distances. Field testing also established the variance in TA for repeated measurements at fixed distances, critical to assessing fix accuracy. Having observed TA-parameter behavior in a simple fielded network, computer simulation was used to extend these results to multi-BS networks, establishing fix accuracy for geolocations based on the TA ranging parameter in WiMAX networks.



Figure 2. Illustration of Overlapping Timing Advance Range Rings.

## **E. THESIS ORGANIZATION**

Chapter II begins the exploration of the 802.16 standards. Detailed analysis of the initial entry procedures specified in the 802.16 standards, as used in WiMAX devices, indicate the windows of opportunity to observe TA packets and what information is expected to be contained within them. Laboratory testing verifying the ability to capture the needed packets from the air interface is then examined in Chapter III. After having established the expected TA parameters and verified the ability to observe them over the air interface, Chapter IV develops the computer tools and supporting mathematics to establish geolocations based on these parameters.

With the background of packets and methods established, Chapter V sets out for field testing, describing both experimental procedures and results of trails conducted from short ranges to distances extending to 1.3 km. Excellent linearity and low variance are observed, which establish the statistical parameters used in further simulation. Chapter VI takes the results from Chapter V's field testing and applies them to several Monte Carlo simulations, establishing the expected fix accuracy in real-world WiMAX network implementations. Finally, Chapter VII pulls together these results and recommends potential avenues for further exploration and refinement of this method.

## II. WIMAX WORKINGS

The 802.16 standard incorporates many features making it appealing on different levels. From the beginning, it is a convergent technology designed to work with internet protocol (IP) applications at the upper levels of the protocol stack, allowing for both voice over IP (VoIP) and data transfer. As more telephony shifts toward VoIP transmission over the backbone networks, this allows for seamless interoperability all the way to the handset. At the same time, while current schemes to move data to cell phones are adaptations shoehorned into what was designed to be a voice channel, shifting to a converged IP architecture again allows for more seamless integration of the handset into the larger data network. At the physical layer, 802.16 allows for the use of many different parts of the spectrum offering, and WiMAX provides for a flexible subset. Variable-encoding schemes are available depending on channel conditions, and the use of orthogonal frequency division multiplexing (OFDM) provides high throughput and robustness against multipath issues.

OFDM may, at the onset, seem somewhat intimidating, but can essentially be thought of much like traditional frequency division multiplexing. Different frequencies carry different pieces of information, just as different radio stations have different music on different frequencies. In OFDM, one radio station just works with a group of frequencies rather than a single carrier. By subdividing the allotted bandwidth to use carefully spaced frequencies, multiple symbols amalgamated via Inverse Fast Fourier Transform (IFFT) can be sent simultaneously. Through calculated selection these sub bands can be spaced orthogonally, avoiding any inter symbol interference (ISI).

Beyond tightly packing many carriers in the allotted bandwidth, these narrower frequency bands are in turn wider in time. Symbols that are wider in time are much less susceptible to time-smearing effects of multipath propagation, where the signal is received as many different modes and reflections. There is of course a trade-off in that the narrower the frequency bands become, the more susceptible the signal is to Doppler shift, and frequency shifts can lead to the nulls failing to align, creating significant ISI.

As an example of the frequency subdivision of OFDM, our fixed WiMAX test equipment uses a bandwidth of 3.5 MHz divided by an IFFT size of 256 (a table of WiMAX profiles is included in Appendix A) [10].

Given OFDM's relative time-robustness, one may wonder why an accurate timing-adjustment mechanism is needed and included in the standard. The answer lies in the media access (MAC) layer, between the physical signaling implemented via OFDM and higher IP functionality. In order to control access to the shared wireless medium, a time division duplexing (TDD) scheme is used where first the BS transmits its information during the downlink before SS are allowed to transmit their information during allotted times during the uplink. The BS acts as controller, scheduling and granting access to certain uplink bursts to specific SS in order to maximize throughput in a contention free manner. To initially enter a BS's network, there is a contention channel and time window for SS to submit their requests, but once established, the BS sends out both a downlink and an uplink map, telling the SS when to listen for their information and when they are free to send.

In order to keep this process functioning in an orderly manner, it is important for each SS to transmit their data at the correct time for it to arrive at the BS in its designated uplink slot. Timing for the downlink is established since all stations use the preamble at the beginning of the frame as a timing reference and simply count slots back from the beginning of the frame. Any delay in the downlink arriving at SS is then irrelevant since the frame clock starts when the frame arrives. However, on the uplink, since the data the SS is sending is still part of this frame, propagation time must be accounted for so that the SS's information arrives at the BS in the correct slots as assigned in the uplink map. Because of this TA is a critical component of ranging.

## **A. NETWORK ENTRY AND RANGING**

As previously mentioned, in order to join a BS a SS must compete in a contention window, be recognized by the BS, and assigned frame slots. When initialized, a SS scans its allowed frequencies (as provided by the service provider's network) to see if a WiMAX network is available. Once a BS frequency is acquired, the SS listens for the

frame preamble to synchronize itself. After preamble synchronization, the first thing the BS transmits is the frame control header (FCH) followed by a downlink burst containing broadcast messages. The FCH and this downlink contain information on modulation, the downlink and uplink maps, and channel descriptors. Figure 3 illustrates the sequence of this data occurring in the WiMAX frame. Based on this information, the SS can determine whether this BS frequency will suitably accommodate it or if it needs to continue scanning.

If the SS finds the BS frequency suitable, it will also be able to find the contention opportunities for initial ranging. During this contention slot for ranging, a ranging code, selected from a known set of pseudo noise (PN) codes, is modulated via binary phase shift keying (BPSK) and transmitted over consecutive OFDM symbols specially appended without phase discontinuity. BPSK provides the greatest robustness of the modulation schemes available in WiMAX and the PN codes allow the BS to separately detect a SS if a collision occurs during initial ranging. If the SS does not receive a response from the BS after a certain time, it assumes ranging was unsuccessful, and begins back off to enter a contention resolution phase before reattempting entry.

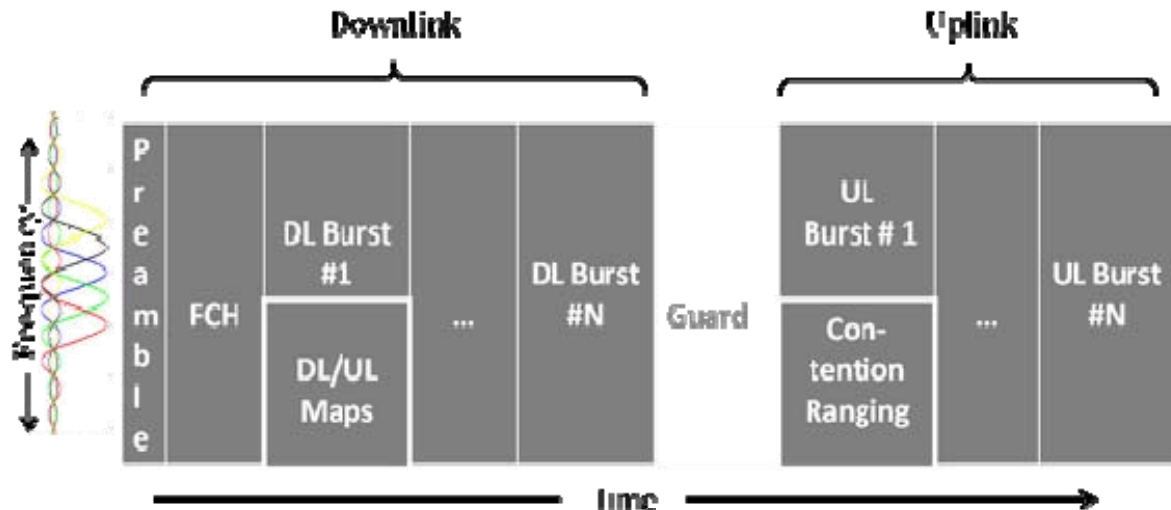


Figure 3. WiMAX Frame Format.

If the BS properly receives the initial ranging request (RNG-REQ), it responds with a ranging response (RNG-RSP) message during the next downlink, which indicates to the SS any adjustments needed to its power, frequency, and timing. This initial response contains the MAC address of the SS for identification purposes, but also assigns a connection identifier (CID) which will be used to address the SS in further traffic. The SS will again transmit a RNG-REQ during its uplink slot and receive either a RNG-RSP message for further adjustments or a RNG-RSP indicating the ranging status is complete. The ranging handshake is shown in Figure 4.

Following initial ranging, network entry continues by negotiating services, authenticating and registering with the network, obtaining an IP address, and obtaining other parameters as illustrated in Figure 5 [10].

Beyond network entry's initial ranging, ranging also occurs periodically to account for changes in channel conditions and mobility, during bandwidth requests, and for handovers. Handover ranging is particularly interesting for geolocation since the message contains ranging from multiple BSs [11].

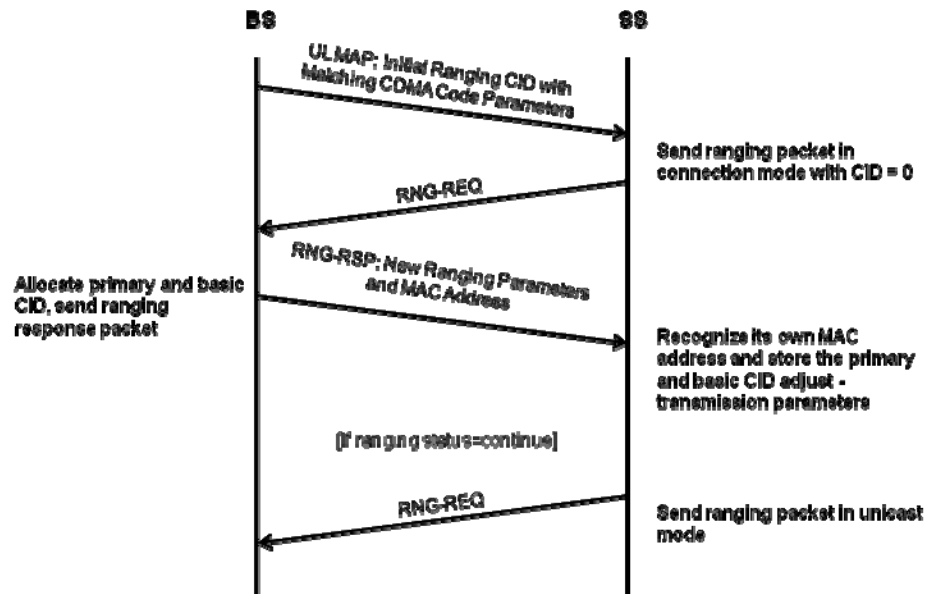


Figure 4. Ranging Procedure (After [10]).

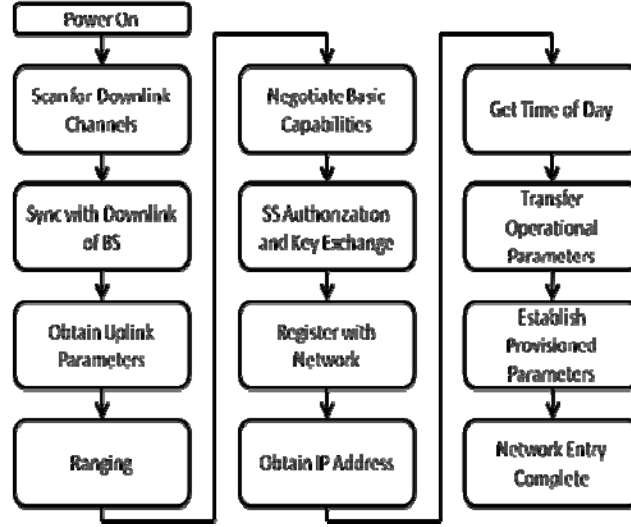


Figure 5. Network Entry Process (From [10]).

## B. RANGE RESPONSE MESSAGE

The RNG-RSP message is defined in the 802.16d-2004 and 802.16e standards to contain a 4-byte TA containing a 32-bit signed number; negative to advance the burst transmission time and positive to delay [11],[12]. The standard defines the value to be variable depending on the physical layer, but commonality is expected between the fixed and mobile WiMAX standards based on nearly identical verbiage in the 802.16 standards shown in Table 1 and the focus on vendor TDD interoperability [4].

<b>802.16d-2004 (Fixed)</b>	Tx timing offset adjustment (signed 32-bit). The time required to advance the SS transmission so frames arrive at the expected time instance at the BS. Units are PHY specific (see 10.3).
<b>802.16e-2005 (Mobile)</b>	Tx timing offset adjustment (signed 32-bit). The amount of time required to adjust SS transmission so the bursts will arrive at the expected time instance at the BS. Units are PHY specific (see 10.3). The SS shall advance its burst transmission time if the value is negative and delay its burst transmission if the value is positive.

Table 1. TA Definitions from 802.16 Standards (After [12],[11]).



The expected consistency indicates that work on geolocation based on fixed WiMAX equipment, which is significantly less expensive and more readily available for laboratory experimentation, will be easily transferable to mobile WiMAX. Vendor TDD interoperability implies that observations on one vendor's WiMAX equipment will also be a valid representation of other vendors implementations.

### C. TIMING ADJUST

Further refining the expected range per unit of TA, the PHY specific units as described in Table 1 are simply the reciprocal of the sampling frequency. The sample frequency is defined in the standard as:

$$F_s = \text{floor} (n \cdot BW/8000) \times 8000 \quad (1)$$

The sampling factor,  $n$ , is dependent on the bandwidth used and also specified in the standard. Appendix B contains a table of OFDM symbol parameters for 802.16d-2004, including an enumeration of the possible values of the sampling factor. For the utilized bandwidth of 3.5 MHz, the sampling factor is 8/7. Solving for  $F_s$  with this sampling factor and a 3.5 MHz bandwidth gives 4 MHz. Using the speed of light, approximately  $3 \times 10^8$  meters per second, and a TA of  $1/F_s$ , each unit of TA should correlate to a distance of approximately 75 meters.

This chapter has thoroughly examined the specifics of the WiMAX entry process as well as addressed specific parameters of interest. Initial ranging will occur at a predefined time in the uplink window, and responses from the BS will include both the SS MAC address and the adjustment instruction. TA, which is documented to be the same in both fixed and mobile implementations, is able to be resolved to distance allowing for a radial distance from the BS to be calculated which can be applied to geolocation by means of crossing radii as introduced at the end of the previous chapter. Having detailed network entry and bit specifics from the standards, the next chapter documents laboratory collection confirming the captured and decode signal is what is defined and expect from the standard.

### **III. LABORATORY OBSERVATIONS**

#### **A. INITIAL OBSERVATIONS IN THE NPS NETWORKS LAB**

To begin exploring the possibilities of geolocating a WiMAX SS based on TA values, as had been explored with GSM's timing advance [8],[9], traffic was first analyzed to ensure RNG-RSP messages could be identified and the necessary information was discernable as suggested by the standard. A small WiMAX network was configured in the laboratory. The network consists of a Redline AN-100U BS and two Redline RedMAX SU-O outdoor SS. The AN-100U was configured to use a center frequency of 3.40175 GHz with a 3.5 MHz bandwidth via network interface and in the laboratory connected to a laptop hosting a file server application. The SS are simply connected to 120 V wall power without need for further configuration and, when attached to other laptop terminals, sustained network traffic could be generated.

Collection of the air interface is achieved by an antenna situated between the BS and SS, which feeds an Agilent 4440 Spectrum Analyzer and a Sanjole WaveJudge 4800 WiMAX analysis box. The WaveJudge is a passive protocol-analyzer which provides protocol and higher-layer capture and decode capabilities and can correlate RF to MAC data. Focusing on the MAC, the WaveJudge was the primary observation instrument, able to capture and decode up to eight seconds of OFDM symbols and display the results to a computer via universal serial bus (USB) data transfer. The limited capture time shaped initial observation in limiting observations of the RNG-RSP to initial ranging, and no periodic ranging was identified.

Viewing the decoded traffic, the RNG-RSP was observed for repeated trials at a range of 10.5 meters, the length of the lab room. Table 2 shows a sample of the actual bits in one of the RNG-RSP messages, illustrating the time length value format of the packet, with information type and data length indicated before the data bits.

### Management Message Type 5, Ranging Status 1 "Continue"

	Type	# Bytes	Value
<b>Timing Adjust</b>	01	04	FF FF FF BE
<b>Power Level Adjust</b>	02	01	EB
<b>Offset Frequency Adjust</b>	03	04	00 00 00 7A

Table 2. Sample RNG-RSP Values.

Having successfully identified and verified the RNG-RSP contained a fairly consistent TA at a static range, the SS were moved to assess the variability and resolution of the TA at different ranges. Initial attempts to move both the BS and SS for maximum range deltas were unsuccessful due to in-lab transmit power limitations and the directional properties of the laboratory collection antenna precluding capture of traffic with the BS relocated. Since only the downlink was of concern to capture RNG-RSP messages, and RNG-REQ were less significant, the BS was left static in its original lab location, and only the SS were moved. Later testing with greater ranges would provide more amplifying data, but as other students were working on calibrated measurements, disruption of the collection equipment was not yet feasible.

RNG-RSP observations varying the distance to the SS provided initially promising results. Figure 6 shows the TA associated with range in meters. Of note, repeated values at a given range cluster since the TA only takes on discrete values. Overall, the results showed high correlation between range and TA, and the TA for repeated trials at fixed range showed a standard deviation of 0.79, meaning most of the time the TA was plus or minus one unit.

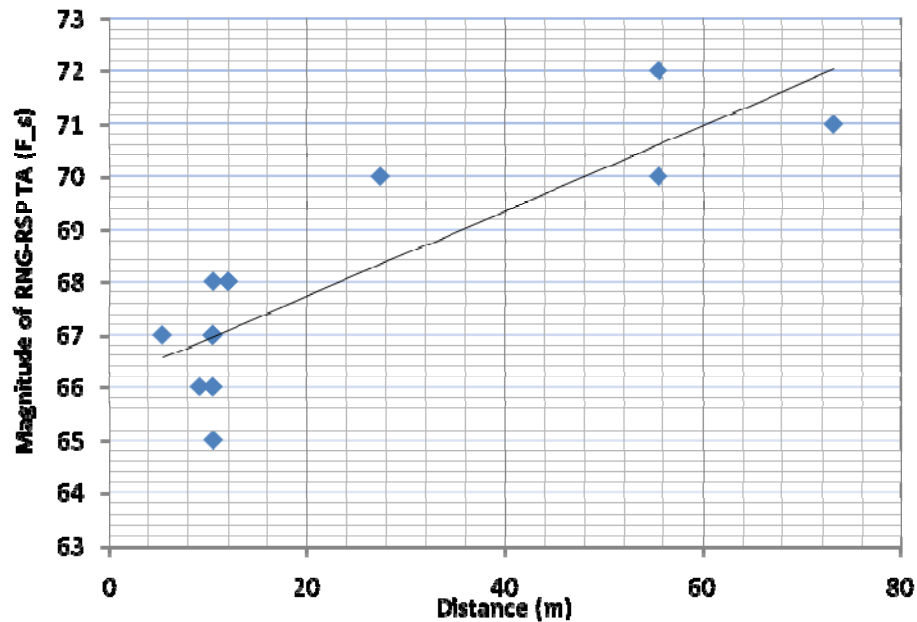


Figure 6. Laboratory RNG-RSP TA vs Distance.

Notably, at distances beyond 12 meters the SS were moved out of the laboratory where the BS was located to a passageway. This configuration no longer provided line of sight between the BS and SS and introduced more multipath in the RF link. Considering this and the rather limited distances within the building, rather than simply using the data trend line illustrated in Figure 6 to approximate change in TA with distance, the data could also more broadly be interpreted to represent one cluster in the laboratory and one cluster in the passageway. This still indicates very consistent results at similar distances, with greater distances resulting in greater initial TA.

Finally, the observed TA were all negative, which recalling from the standards, is defined to indicate the SS should be transmitting sooner. A larger negative number indicates the SS should be making more and more correction to transmit earlier in time. For the sake of clarity, all remaining discussion of TA during the initial ranging will simply indicate the magnitude of the TA, so a great TA will reflect greater distance and timing advance, avoiding any possible confusion with larger negatives being smaller.

## **B. OBSERVATIONS OF TEST DATA FROM SANJOLE**

In the process of optimizing test methodology with the WaveJudge, during a site visit and meeting with Sanjole Chief Technology Officer, Dr. Xavier Leleu, several other capabilities of the WaveJudge were discussed beyond simply decoding the traffic from the air interface. Sanjole had designed the WaveJudge to provide diagnostic capabilities to equipment vendors after witnessing difficulties observing intersystem communications at PlugFest events. In these settings, testing with the WaveJudge is often conducted over wired channels, rather than the air interface as in the NPS laboratory.

Two interesting features that might be useful in automating geolocation were also presented. The WaveJudge has scripting capability, which would allow specific parts of the decoded WiMAX messages to be passed to another program over a TCP port. However, use of this scripting feature requires additional licensing, and even using the scripting feature, the WaveJudge still transfers the entire base band signal to the analysis computer, creating a limited collection window based on available memory and introducing extra delay during processing.

Another feature of WaveJudge is the ability to range certain packets. During a bandwidth request, a WiMAX subscriber transmits one of 64 known PN codes. Just as the BS can range the SS based on known codes during network entry, the WaveJudge can find the range from its antenna's location to the SS based on delay in the reception of the PN code from the SS's assigned transmission time seen in the uplink map.

Observing such ranges in Sanjole's data from wired testing with the WaveJudge logically collocated with the BS, the equivalent TA seen from the WaveJudge was consistent with the BS's TA in the RNG-RSP with the exception of a fixed offset. Leleu reported that an offset between the transmit and receive channels has been observed in most BSs, causing an offset value that the link is able to adjust for during the ranging process by simply adjusting for it in the TA [13]. This agrees with and explains the offset value seen in the initial collection in the NPS laboratory.

Ranging based on known PN codes from a collection and analysis box, such as the WaveJudge, could easily add a second location from which to establish a radius to aid

in the geolocation process. However, while the initial RNG-RSP contains a MAC address, responses to the PN code are simply addressed to whichever SS sent that specific code of the 64 known codes, so more associated traffic would have to be stored and correlated to specifically associate which SS was just ranged. This identification issue will have to be addressed for periodic ranging cases as well, since after a connection identifier is established, the network no longer references the MAC address. For the duration of this paper, we will continue to focus specifically on ranging based on the RNG-RSP.

This chapter began examining actual collection of the network entry process from the air interface and illustrates a sample of actual time-length-value encoded bits decoded from RF. This collection verified consistency with the standard, and collection within the confines of the laboratory facilities began to show highly consistent values with linear correlation to distance. Insights from the equipment vendor confirmed a timing offset can exist between the BS send and receive channels, as well as highlighting the potential to establish a range from the collection platform and the ability to use scripts to pipe timing-data into other programs. With this ability to rapidly and automatically move the decrypted TA to another program in mind, the following chapter explores the development of a user interface as well as the underlying mathematics to calculate a geolocation based on this timing data.

THIS PAGE INTENTIONALLY LEFT BLANK

## **IV. INTERFACE DEVELOPMENT**

### **A. GUI BACKGROUND**

In order to facilitate the practical use of the method of crossing range radii from two or more BSs or a BS and other collection sites capable of ranging based on known PN codes, automated computation accessed through a graphic user interface (GUI) best enables an emergency responder or tactical user without requiring in depth technical background or complicated calculations. The use of any automated system requires user understanding of its capabilities and limitations. While a GUI rapidly presents a location approximation, it is important to remember that there is variation in TA measurements and output locations represent a probable area and not certain point. The approximations are also only based on the programs internal algorithm and the user may have other information that further refines an accurate location approximation or refutes an errant estimate. For instance, in a case where there are multiple intersections between two radius rings, a computer will be unable to differentiate them, but other situational information may cue a user to favor one intercept as having a higher likelihood of being the SS's actual location.

Such a GUI was developed in HTML with JavaScript, utilizing the Google Maps API to provide access to global satellite and terrain maps. The web interface allows for easy cross platform use without the need for custom hardware or map databases. The general layout of the GUI consists of a simple form to accept BS and collector locations with TA information before rendering the plot with site coordinates, range radius rings, and a likely SS location ellipse. It would be possible to populate these fields from a database containing known BS locations, a GPS position of the collector, and scripted TA output feeds from an analysis device like the WaveJudge. For purposes of testing and evaluation, the GUI also contains fields to input the known location of a SS so its true location can be compared to the output approximation during trials. While the details of the HTML implementation and syntax used with the Google Maps API are beyond the scope of this paper, the complete code is included in Appendix C. The JavaScript section



of the code contains the mathematical implementation of the geolocation approximation and the employed methodology is examined in detail below.

## **B. ENABLING APPROXIMATIONS**

Utilizing a method of intersecting range radii based on propagation delay acquired from signal internals, a number of initial working parameters were first established. As discussed in Chapter II, assuming free space propagation at the speed of light, approximately  $3 \times 10^8$  meters per second, at the bandwidth used each unit of TA seen from the BS increases the range radius by 75 meters. This provides the basis for all range radii in the calculations, both limiting resolution to 75 meters in the best case scenario and accepting that there are variance and deviation in measurement.

A further initial approximation is the use of a flat Earth to facilitate calculations in Cartesian coordinates. While a flat Earth approximation does not introduce significant error over typical cell ranges, which are only infinitesimally curved on the geode, it simplifies calculations of range radii, intercepts, and probability polygons to work in a meters-by-meters coordinate system. However, mapping the results back to the spherical system of latitude and longitude on Earth's surface requires a coordinate transformation.

Over the entire globe, parallels of latitude are all parallel with the equator and effectively equally spaced. However, meridians of longitude converge at the poles. At the equator, a degree of longitude is roughly equal to a degree of latitude. However, moving toward the poles, the meridians converge so the distance per degree of longitude diminishes while the distance per degree of latitude remains consistent. While this does not affect the Cartesian calculation - ten meters by ten meters is the same no matter the location - it does introduce some complication into mapping the results calculated on a square grid back to the surface of the Earth.

As a simplifying assumption, it is approximated that over small distances the convergence of meridians is negligible, so meridians of longitude are parallel. This is analogous to the far field approximation generally employed in radio frequency analysis that as the radius of curvature becomes exceedingly large at a distance from the transmission source it can be modeled as a plane wave. Assuming both lines of latitude

and longitude to be self parallel, they form a rectangular grid. The results of Cartesian calculations can simply be mapped from a square grid to a rectangular grid by multiplication by a linear constant in each direction, drastically simplifying in implementation without appreciable loss of fidelity.

A degree of latitude is approximately 110 kilometers, and the length of a degree of longitude is calculated based on scaling this value at the equator by the cosine of the latitude, resulting in equivalent distances at the equator and the length of a degree of latitude collapsing to zero at the poles. While these approximations provide very accurate results in most cases, very near the poles some anomalies may manifest in this coordinate system mapping.

A final coordinate mapping concern is the orientation of the angles. In most two-dimensional mathematical applications, the X axis is horizontal with the Y axis vertical. Zero degrees is defined to be straight to the right in the positive X direction, while a positive 90 degrees is counterclockwise a quarter revolution, straight up along the Y axis. However, working on the globe, it is customary to define north as zero degrees, while 90 degrees is defined by turning clockwise to the east. This requires a 90-degree shift and then an axis flip to overlay the coordinate systems. All of the trigonometric sine function calculations still work on the flat Earth projection, but axis orientation must be taken into account to maintain the proper reference frame.

### **C. LIKELY LOCATION CALCULATIONS**

Given two stations, each with a known radius, the most likely location of an SS is where the radius rings overlap or are closest to overlapping. Broadly, three basic situations can occur: the radius circles can intersect, one radius circle can be completely contained within the other, or the two radius circles can be separated without touching. With only two stations, overlapping rings results in two intersections, one of which is the coordinate solution to the most likely SS location based on those rings. A third station will almost always remove the ambiguity, since all three rings only converge near one of the two intersections of the two ring solution. Figure 7 illustrates the decision process as a flow chart.

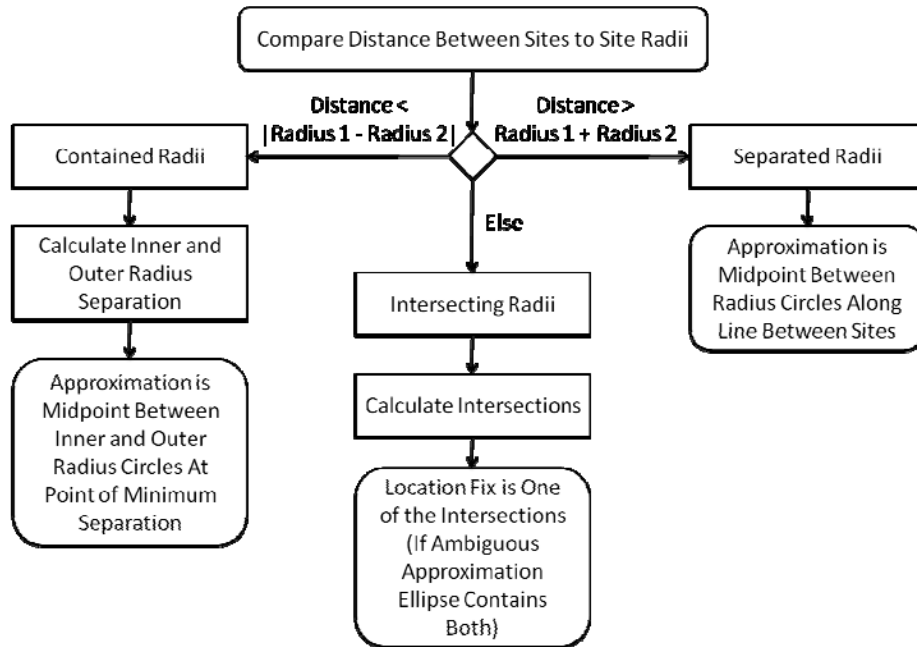


Figure 7. Flow Chart for Geolocation Method.

Initially limiting the number of range rings to two, based on available test equipment, the best possible location estimate will occur as the two intersections get closer and closer to the point where the radius circles only touch in one point. Given this best fix is achieved with the radius circles barely touching, concentric or separated circles also provide the possibility of an accurate approximation when they are close to touching. Two radius circles that come very close but do not intersect is as valuable a solution as a single intersection, but obviously the further apart the nonintersecting radii become the less and less meaningful the result.

In order to determine which of the three basic cases has occurred, the algorithm generalized in Figure 7 compares the distance between the sites, solved by the Pythagorean Theorem based on their grid coordinates, to the sum and difference of their radii. If the two radii combined do not sum to the total distance between sites, there is no intersection, and the estimated location is simply the midpoint plus an approximation radius based on a scaling factor and the distance between the station's radius circles.

Alternately, if the distance between the sites is less than the difference in their radii, one radius circle is completely contained within the other. As long as the sites are

not exactly collocated, in which case no direction information can be ascertained, drawing a straight line through the sites on one side will result in the maximum separation and on the other side the minimum separation where the radii circles are closest together. Again, the approximation area is the midpoint where they are closest together plus a radius scaled by a constant at the separation distance.

If the radius circles are neither self contained nor completely separate, then they intersect. One approach to find their intersections would have been to iterate through all the points used to draw the radius circles on the map and find the ones that are closest together based on a tolerance for rounding and the limited number of points used to approximate a circle. More accurately and directly though, the method used to find the intersections is based on triangles. The geometry of this method is illustrated in Figure 8.

The distance to the middle of the circles overlapping can be easily determined from the law of cosines based on the site radii and total distance. To derive this, the Pythagorean Theorem is simply applied to both of the triangles, knowing each hypotenuse is the radius and the base is the total distance between sites less the distance from the midpoint to the other site. Since both triangles share the last side, solving each triangle for this side, the two equations can be set equal and solved to find the distance from either site to the midpoint.

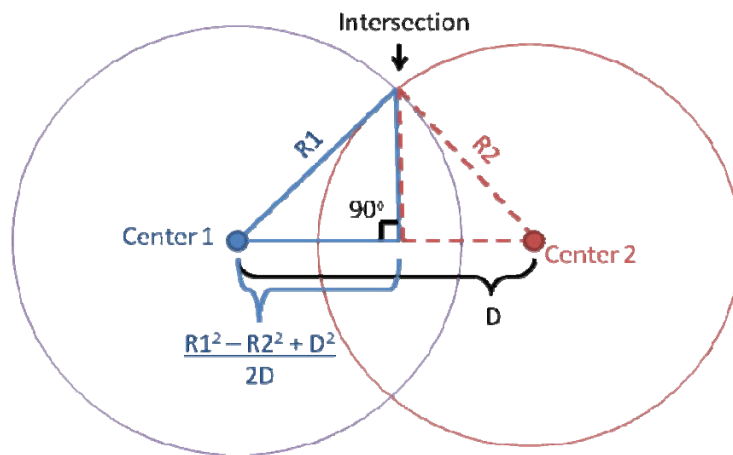


Figure 8. Illustration of Geometry to Calculate Circle Intersections.

Both intercepts lie on a line perpendicular to the line between the sites through this midpoint. Having calculated this base distance, and knowing the hypotenuse of the triangle formed from a site, the midpoint, and an intersection is simply the radius of that site, the triangle can be completely solved to find the distance from the midpoint to the intercept that was eliminated in the previous derivation when the equations were set equal. Having completely solved the triangle, the intersection location relative to either the midpoint or the site can be calculated, needing only to take into account the coordinate system constants so they plot at the appropriate latitude and longitude on the map.

As an example, working in Cartesian coordinates with one BS at the origin and the other a distance  $D$  along the  $X$ -axis, the  $X$ -coordinate of both intersections is calculated from the radii,  $R$ , and inter-site distance,  $D$ :

$$X = \frac{R_1^2 - R_2^2 + D^2}{2D} \quad (2)$$

The  $Y$ -coordinates of the intersections would then be found via the Pythagorean Theorem to be:

$$Y = \pm \sqrt{R_1^2 - \left( \frac{R_1^2 - R_2^2 + D^2}{2D} \right)^2} \quad (3)$$

While the likely probability ellipses for both contained circles and separate radii are simply plotted as circles, in the intersection case an ellipse is drawn around the two intersections. In a case where the correct intersection representing the SS can be identified, the probability ellipse can be viewed as the area of overlap from variance-wide TA bands. This variance value is refined in field testing, but for initial GUI implementation with intersection ambiguity, the ellipse is plotted to contain both intersections with the major axis is based on the distance between the intercepts and the minor axis is based on the separation between range radii. The center of the ambiguous two BS ellipse is the same midpoint calculated to solve the intercept positions and the relative angle of the intercepts defines the tilt of the ellipse (this is simply 90 degrees from the angle between the sites). Separate cases deal with whether the intersecting

circles centers both contained within the larger radius, overlapping from within, or whether the two circles overlap from the outside to provide the most reasonable approximation.

#### **D. DISPLAYED RESULTS**

After the calculations, the Web interface displays the map with both stations displayed; their range radius rings, and the likely location ellipse polygon. Mousing over the respective icons identifies the site and provides its range radius in meters. If a SS coordinate was entered, it will also be displayed and have its distance to the center of the approximation polygon in its mouse over popup. If no SS coordinates are entered, the SS defaults to the ocean at equator and the prime meridian, effectively eliminating its display in most cases at useful zoom levels. None of these icons display latitude and longitude coordinates since it is already displayed at the top of the web page and would only clutter the map.

Checking the details button provides an alert popup containing more information about the processed calculations and causes the map to also display the ellipse center point and the line connection the base sites. The API allows the Google Map to continue to behave as a user would expect under these overlays so a user can pan and zoom the map, switch between street maps, imagery, or terrain maps, and re-center the map by clicking on the hand in the upper left.

While database lookups and scripted input may be future additions, even hand entering observed values from the WaveJudge provides a baseline functional tool. The sample import button simulates what might be able to be scripted to import by populating fields with predefined values contained in the HTML for a test scenario in Monterey. The user interface, after a run with this simulated import, is shown in Figure 9.

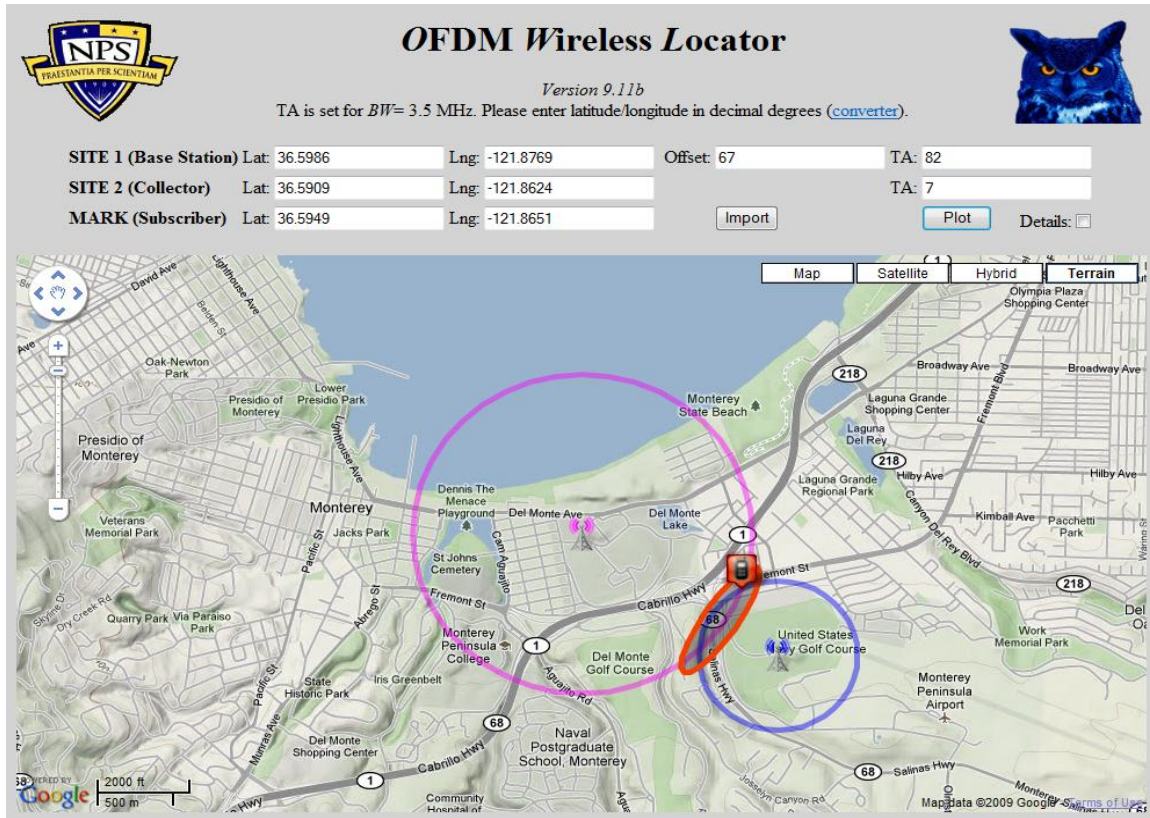


Figure 9. Sample GUI Screen Shot.

Having discussed the need for a GUI, this chapter addressed fundamental mapping issues including scaling and coordinates transformations to present data on the globe's grid of latitude and longitude rather than simply in generic two-dimensional Cartesian space. After discussing these basic issues, the details of the actual geolocation calculation were discussed. There are three scenarios in which radii from two BS can be used to find a location approximation dependent on the distance between the BS and the radii established by their TA. In the case the radius circles intersect, the mathematics of how to find the intersections of two radii was shown. These simple principles can be extended to greater numbers of BS further refining geolocational accuracy, but first, equipped with a GUI that can easily run on a laptop and facilitate near real time geolocation from a collection vehicle, further field testing is conducted to measure the distance seen per unit TA and variance in observed TA at fixed locations over real world distances.

## **V. FIELD EXPERIMENTS AND RESULTS**

Following basic lab testing to verify that the RNG-RSP packet could be captured and having created a preliminary web-based program to estimate locations based on radii from two known points, field testing was conducted. There were two main goals in field testing. First, confirm that the TA captured in the RNG-RSP continued to linearly correlate to distance between the BS and SS over several samples at more practical real-world distances. Second, testing sought to establish variability in repeated measurements at a fixed point. This variability in repeated measurements is crucial in determining the accuracy of a location fix based on TA, since a high variance in TA at a fixed location would lead to a rather large area of uncertainty in approximating a SS location.

### **A. TESTING LOCATION, CONFIGURATION, AND PROCEDURE**

In order to conduct testing, the same Redline WiMAX equipment used in the lab was used to establish a simple outdoor WiMAX network, consisting simply of the BS and SS operating at 3.4175 GHz configured the same as in the laboratory. Since this WiMAX frequency is in the licensed band, full scale testing could not be conducted on and around the NPS campus in Monterey, so instead arrangements were made to conduct testing at Camp Roberts, outside of San Miguel, CA.

The testing site around McMillan Airfield at Camp Roberts was predominately flat terrain with some trees and small single-story buildings, providing primarily line of sight conditions during testing. Conditions were clear, with temperatures approaching 35 degrees Celsius at midday. The only other signal in the vicinity of the WiMAX operating frequency was an approximately 2-5 GHz spread spectrum signal creating negligible interference.

Configuring the WiMAX network at Camp Roberts, the AN-100U BS and antenna were mounted on top of a raised observation platform, with the antenna mounted 7 meters above the ground. The exact same cabling and settings were used as in the lab, and the BS was powered up and left alone for the remainder of the testing. The SS was



attached to a portable wooden stand with the SS antenna at 2 meters above the ground. Neither the BS nor SS were connected to other computers or data sources.

Network entry was facilitated simply by powering up the SS, which began the network entry process, just as turning on a cell phone negotiates network entry even if the user is not immediately placing a call. Powering down the SS and powering it back up at the same location allowed for repeated captures of the network entry process to capture multiple RNG-RSP messages at the same distance from the BS.

Collection in the field was done from a vehicle, simulating what would realistically be done by emergency response personnel or tactical operators. Using a roof-mounted omni-directional antenna, the WiMAX signal was captured from the air interface by the WaveJudge. Manually clicking to open the RNG-RSP response packet in the WaveJudge interface, the TA was extracted and recorded. GPS coordinates were also manually entered from the output of a basic consumer Garmin nüvi 200. More advanced scripting may still allow this data to be exported directly from the WaveJudge and a GPS device to feed a geolocation program like the one developed in the previous chapter, but manual data entry was still timely during the experimentation. Figure 10 diagrams the configuration of test equipment. Photographs and a sample WaveJudge screen capture are included in Appendix D.

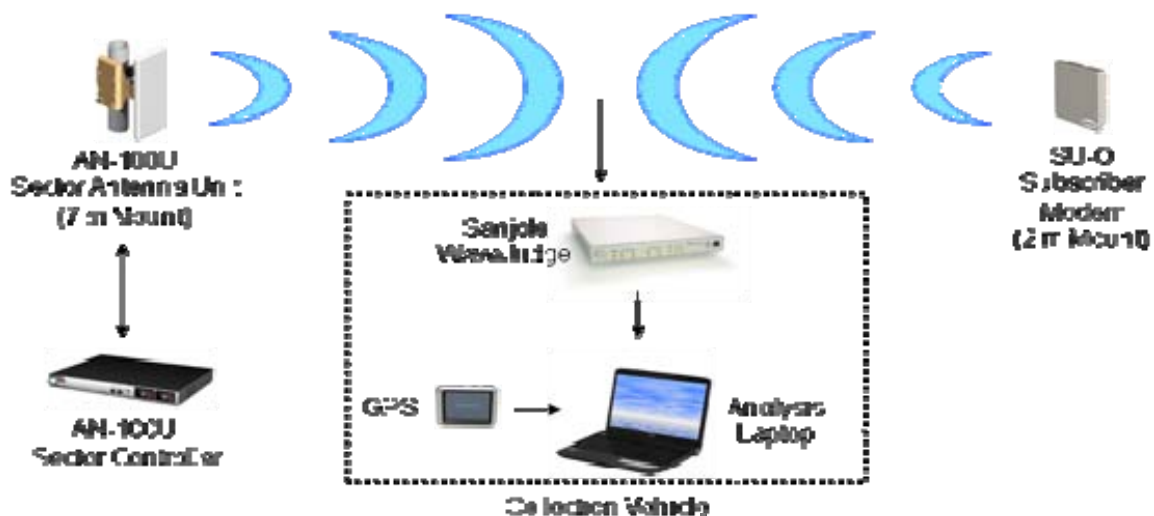


Figure 10. Test Configuration.



Figure 11. Field Collection Station Locations.

The SS was set up at 6 different locations as shown in Figure 11, with trials run until at least 5 TA were successfully collected. One exception occurred later in the day at the final range of greater than 1 km when only one TA was successfully captured due to equipment difficulties. Initial observations showed very high consistency between repeated observations at fixed distances and what appeared to be predictably greater TA with increasing distance.

## **B. NOTED CHALLENGES DURING FIELD TESTS**

Several challenges presented themselves while working with the WaveJudge as a piece of collection equipment. Sanjole designed the WaveJudge to provide vendor laboratory analysis to aid with design and interoperability testing [13], not provide real time output. As such, the entire collected OFDM waveform is shifted to baseband and transferred to the analysis computer for decoding, rather than being broken out to the bit stream and sent to the computer. Because of this design, there is a limited amount of time the system can capture before it runs out of memory. During field collection it became

important to time trigger the WaveJudge to cycling power on the SS, otherwise the RNG-RSP was not captured in the limited collection window. In execution, it was found with the tested Redline SU-O SS, triggering collection at approximately 20 seconds after first applying power to the SS fairly reliably captured the RNG-RSP in the eight-second collection window. After a sample was collected off the air interface, the computer still had to decode the base band signal, which introduced further delay into the process.

These issues do not discredit the numerical results or in any way devalue the capabilities of the WaveJudge to capture RF and decode protocols, but we simply point out that in this case the equipment is being used for something other than its initially intended design purpose. To actually field an operational geolocation system, real time processing to the bit stream, as is done by the SS modem, would allow triggering on the RNG-RSP, allowing capture without having to fortuitously align the memory limited collection window.

Another issue that arose in testing later in the day was thermal stress on the BS. Initial collection in clear condition with minimal interference showed minimal errors in any of the packets collected. Later in the day however, more and more errors occurred in the captured streams, not only in the RNG-RSP, but the uplink maps, downlink maps, and many other packets the WaveJudge simply decoded as error packets.

Initially, it was thought that the increasing range may have had an effect, although WiMAX is specified to operate at much greater distances, but using higher gain antennas did not have any effect, and even returning to a distance of less than half a kilometer did not resolve the issue. However, by the end of the afternoon, temperatures at Camp Roberts approached 36 degrees Celsius, and the BS is only specified to operate at up to 40 degrees [14], so operating the WiMAX network on the edge of specifications began to induce errors. If a temperature-controlled space had been available for the BS unit, the antenna did not seem to have any issues, and testing could have continued. However, due to limitations of time and space, this concluded range testing.

Noting the effects of temperature on the BS, while our vehicle maintained functioning temperatures for the collection suite, thermal stress is definitely something to take into consideration in designing systems for fielded operational use.

### C. FIELD TEST RESULTS

Despite the noted issues that arose during collection, the experimentation was successful, and after combing the results of the tests at various ranges, a continued trend of tightly grouped TA with linear progression corresponding to distance from the BS was observed. The collected data is plotted in Figure 12 with results summarized in Table 3. Taking into account a basis offset from the delay between the BS transmit and receive channels and cabling between the BS and BS antenna based on the measurements in the lab, the results show an average meters per unit of TA is 36.99 with a standard deviation of 3.10.

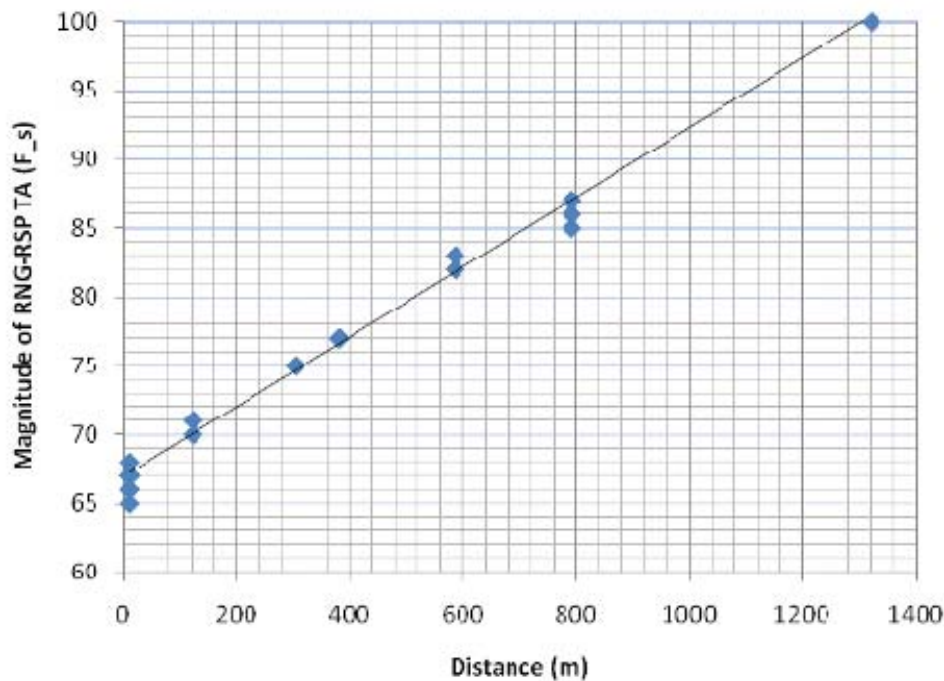


Figure 12. Field Test RNG-RSP TA vs Distance.

BS Lat Dec	BS Lon Dec	SS Lat Dec	SS Lon Dec	Dist (m)	Avg TA	Std Dev	Meters/TA
Lab	Lab	Lab	Lab	11	-66.7	0.786	Basis Offset
35.71574	-120.76390	35.71590	-120.76525	123.5	-70.6	0.548	31.43
35.71574	-120.76390	35.71658	-120.76710	304.5	-75.0	0.000	36.55
35.71574	-120.76390	35.71725	-120.76768	381.4	-77.0	0.000	36.92
35.71574	-120.76390	35.71807	-120.76972	586.5	-82.4	0.548	37.28
35.71574	-120.76390	35.71878	-120.77182	792.2	-86.4	0.894	40.15
35.71574	-120.76390	35.72418	-120.77418	1320.8	-100.0	N/A	39.63

Table 3. Field Collection Results.

Averaging the standard deviation in TA at any given distance gave an average standard deviation of 0.462. To better establish the overall variance in TA, the results of all tests were combined into a single probability density function (PDF) by using the median at any given distance as a reference point as shown in Figure 13. Based on this cumulative data, the overall standard deviation of TA was calculated to be 0.673.

Overall, this reaffirms the initial in lab measurements showing low variance in TA as well as a linear correlation of TA to distance. However, each unit of TA resulting in a radial distance increase of 37 meters is almost exactly half of the calculated distance of 75 meters. This does indicate even higher resolution, and the potential for a more accurate location approximation, but introduces an interesting anomaly because it is more accurate in the case one knows what the actual distance to use is. Given the physical layer dependence of the definition of TA, a consistent ability to calculate it is essential to provide a robust capability against varied fixed and mobile WiMAX networks.

A number of factors could influence the correlation of TA to distance. Over short distances, the difference between measured and calculated TA may be exacerbated by the digital nature of TA. The BS has to round to a whole unit and this may introduce some error. However, at longer ranges, such the 1.3 kilometer distance, this rounding effect will have less impact on the reflected meter per TA. Other sources of channel delay, such as cabling, may also skew TA. Again, however, all cabling propagation distances would have also been accounted for in the initial basis offset and mitigated at greater distances.

Finally, there may be some degree of multipath adding additional propagation time despite the clear, predominately line of sight conditions during testing.

Ultimately, since all of these TA modifying effects would be small and mitigated over greater distances, the factor of two differences between the calculated and measured TA is most likely a phenomenon unique to initial ranging. Recalling from Chapter II, initial range occurs after the SS synchronizes to the received BS preamble. However, since the BS does not know how long it took for its downlink to arrive at the SS, the first delay it sees includes the time it took for the downlink to reach the SS plus the propagation time for the SS's initial ranging request to reach the BS. The total round-trip time takes twice as long as the SS normal uplink to the BS, essentially decreasing the meters per unit TA by two during initial range. However, despite the extra resolution this provides during initial ranging, once the SS has an assigned uplink slot, continuing periodic ranging adjustments may return to a range resolution of 75 meters.

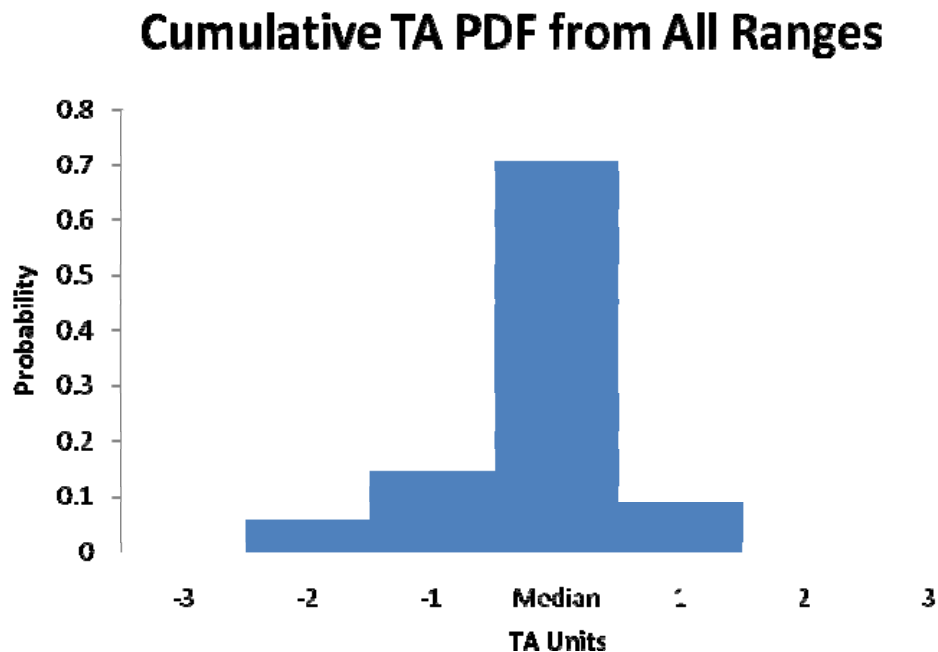


Figure 13. Cumulative TA Probability Distribution from all Ranges.

This chapter discussed the details of field tests conducted to observe TA data over distances more accurately simulating a cellular network environment. Having detailed testing procedures and challenges, results from the field testing showed a very linear progression of TA with increasing distance along with a very low variance in repeated measurements at fixed SS locations, indicating the possibility for tightly bounded geolocation probability ellipses. The range resolution during initial ranging turned out to be twice as good as initial calculations from the parameters documented in the standard had suggested, although this added resolution may be limited to the initial ranging procedure. Having established a baseline for the performance of a standard compliant WiMAX network in-field testing, the following chapter expands the hardware constrained results through computer simulation to provide estimates of geolocation accuracy in varied networks with multiple BS.

## **VI. MULTIPLE BASE STATION SIMULATIONS**

While field and laboratory testing were limited to the one available BS, to better understand the potential for geolocating a SS in a real world WiMAX network, several multiple-BS scenarios were simulated in software. Using the data collected from the BS in the field tests at Camp Roberts, MATLAB was used to simulate multiple BSs with the same mean distance per unit of TA of 37 meter and standard deviation of 0.673 units TA. While this assumes that all BSs in the network share the same characteristics as the one AN-100U BS used in testing, it remains a reasonable premise based on the designed interoperability within the standard.

### **A. TWO BASE STATION SIMULATION**

First, two-BS scenarios were simulated using the algorithm developed in Chapter IV to approximate the location of a SS based on two radii. The simulation created two BSs at varying angles as observed by the SS, each with normally distributed random distance with a mean of one kilometer from the SS, standard deviation in range of 300 meters, and TA fluctuations as observed through measurement. Since TA is a discrete value, the BS first rounded distance to a whole unit of TA, and then on top of this an error factor based on the observed variance seen in measurements was added. A Monte Carlo simulation of one hundred thousand runs were conducted at each angle and the average distance from the center of the approximation polygon (the midpoint between the two circle intersections) to the actual SS location over these many runs was recorded.

At 180 degrees, the two BSs and SS form a straight line, and SS is location is accurately approximated in the middle. However, as the angle collapses from this ideal geometry, the two intersections of the radii get farther and farther apart, and since there is ambiguity as to which intersection represents the location of the SS, the location approximation of a midpoint between the intersections becomes farther and farther from the true SS location. If other a priori knowledge could allow an operator to distinguish between the two intersections, one of the intersections is always close to the SS, but the algorithm alone cannot differentiate the two intersections. Interestingly, the estimate



distance collapses again once the angle between the BS approaches zero, since the BSs and SS are again in a line. Figure 14 shows the overall results of this two BS simulation.

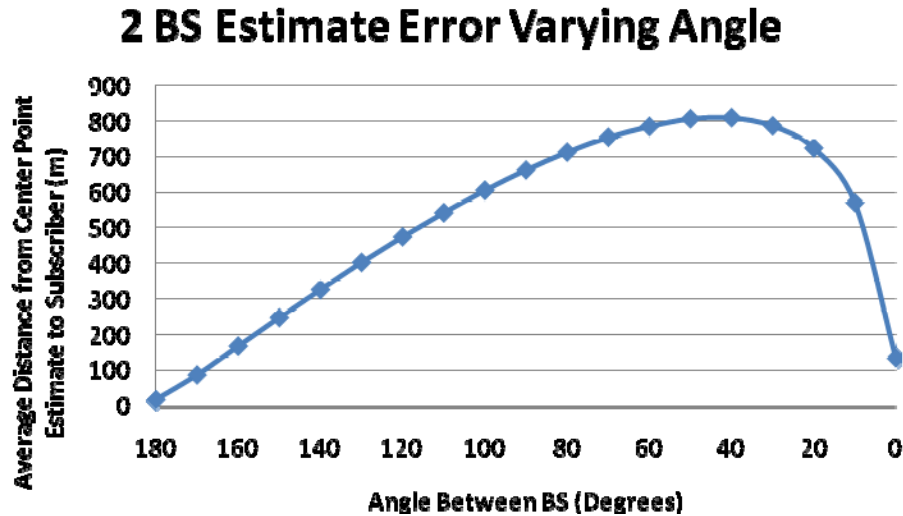


Figure 14. Distance to Midpoint Between Intersections with 2 BS Varying Angle.

## B. MULTIPLE BASE STATION SIMULATIONS

Noting the limitations of a two-BS fix, a second set of simulations was conducted in MATLAB using multiple BS. Except in rare cases introducing a third BS removes the ambiguity between which of the intersections of the two BS represented the approximate location of the SS. Figure 15 illustrates the sort of worst-case scenario that may occur with a three-BS network which generates a very inaccurate location approximation by essentially devolving to approximate a two-BS network.

The general approach of the multiple-BS simulation algorithms is simply to calculate the intersections of each pair of BS radii, and then for each pair's intersections, see which is closest to the nearest intersection of the next pair of BS's radii. This intersection should represent the intersection nearest the SS, and is added to an array of intersections. Once this process has been repeated for all pairs, the array of closest intersections forms the vertices for a small polygon approximating the location of the SS. In the case of three BS, there are six intersections from the three pairs; producing three

chosen closest intersections, which when plotted should form a small triangle near the actual SS location. In order to approximate the center of this polygon not knowing its exact shape or number of vertices, the  $X$  component of each chosen intersection was averaged to find the  $X$  coordinate for the center of the approximation, and the process repeated with the  $Y$  coordinates from the same selected intersections.

Using this location-approximation algorithm with varying numbers of BS, several BS-location scenarios were tested. First, all BS were placed at random angles from the SS with normally distributed random distances with a mean of 1.2 kilometers and a standard deviation of 400 meters. This completely random placement of BS was repeated for 100,000 runs at each number of BSs, and the average distance from the estimate to SS at each number of BSs was recorded as in the earlier two-BS simulation.

As previously noted, an interesting result of the completely random BS placement was that sometimes rare geometries would cause even the three-BS scenario to produce an inaccurate result if the algorithm selected to weight a cluster of intersection where the SS was not actually located. Basically, a three-BS fix can devolve to the two radii ambiguity problem with unfortunate geometry as shown in Figure 15, where triangles show towers, diamonds show selected intersections, the star shows the algorithm's estimated location and the square shows the actual SS location.

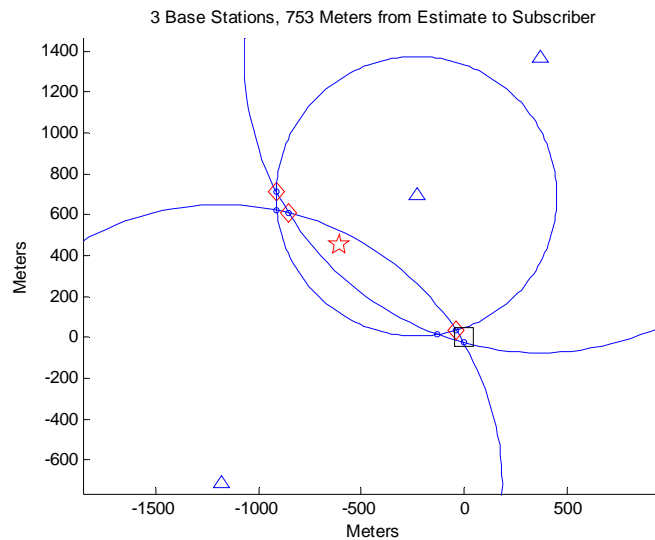


Figure 15. Inaccurate Fix Situation with 3 BS.

Since all of the geometries are averaged in with the other runs, estimates that are hundreds of meters off weigh in with many very accurate estimates from a three-BS geolocation, as reflected in the results shown in Figure 16. Given that the overall average error between the approximated center point and the SS is still only 80 m, there are many more accurate fixes than the outlier cases (such as Figure 15) that pull up the average. Appendix E contains more sample plots from the multiple-BS runs, including a successful three-BS run that accurately estimates the SS location.

In a real world network, BS locations can be anticipated to be more logically distributed. While actual tower locations in modern cellular networks rarely align exactly to the idealized honeycomb cell layout, they are generally spaced to provide maximum coverage with the least number of towers dependent on subscriber density. As such, two follow-on multiple BS simulations were run that added some structure to the BS placement. First, the angles were fixed to be evenly spaced based on the number of towers while distances were randomly assigned as in the initial multiple-BS simulation, and then for even further structure, the same even angle distribution was used but with all BS at a one kilometer range from the SS. Figure 16 shows these results, which show greater location accuracy than the completely random case. In networks with more structured BS spacing the error estimate, despite TA rounding and variations, is still less than 30 meters in all cases.

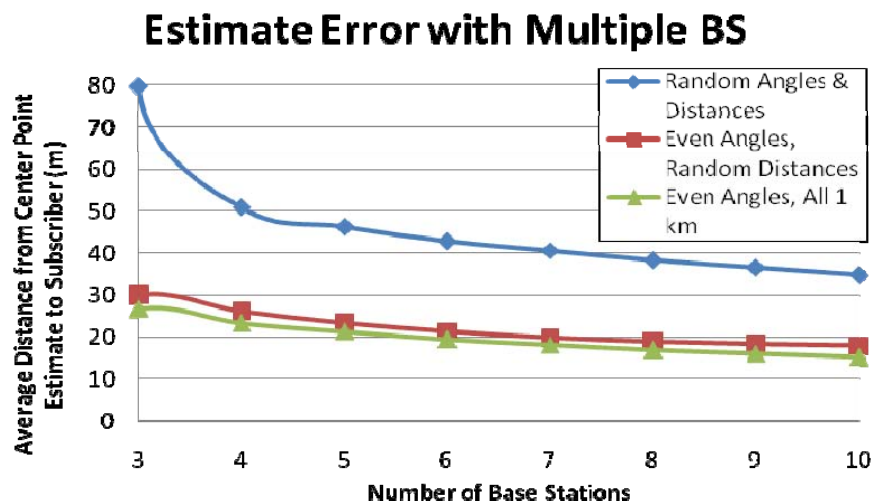


Figure 16. Average Distance from Estimate to SS with Multiple BS.

Based on these results, real-world scenarios can be expected to fall somewhere between ideal and completely random geometries, so a geolocation fix assuming the parameters measured in field testing can on average be expected to fall within 50 meters. Appendix E contains sample plots generated during these different multiple-BS simulations, and in many of these cases, the fix is well within 20 meters.

Having noted the discrepancy between the 37-meter average distance per unit TA observed in initial ranging and the generalized calculated value of 75 meters per TA, the simulations were re-run using 75 m as a basis instead. By increasing the distance between differentiable radii, the resolution is reduced and the effect of rounding error is increased. Figure 17 compares the results of the simulation using 37 and 75 meters per TA. Interestingly, the geolocation accuracy of an ideal-geometry networks with 75 meter TA units is nearly identical to that of a suboptimal geometry in a network with 37 m per unit of TA.

As one would anticipate, the reduced resolution and greater rounding error leads to somewhat larger approximation errors. None-the-less, expecting real-world BS configurations to be between very structured and completely random, it is reasonable to expect an average distance from the approximation center point to the subscriber to be within 100 meters.

As an alternate presentation of this data, rather than using average distance from the SS to estimated location over 100,000 trials, the same scenarios can be accessed via circular error probable (CEP), that is the radius within which 50 percent of the samples lie. The CEP for various numbers of BSs, with both BS geometry configurations and distances per unit TA as before are shown in Figure 18. The same trends are seen as when viewing the averages, but the circle containing 50 percent of the estimates has a smaller radius than the average distance to the estimates calculated.

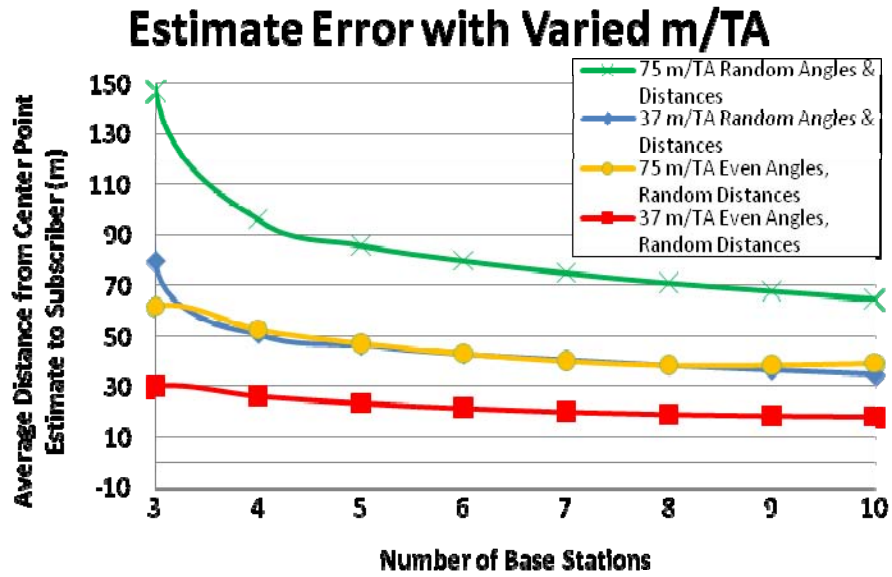


Figure 17. Comparison of Simulations with Different Distance per TA.

Using CEP, as before with averages, it is safe to assume that in more than 50 percent of cases, a SS can be geolocated within 50 meters assuming that accurate BS locations are known.

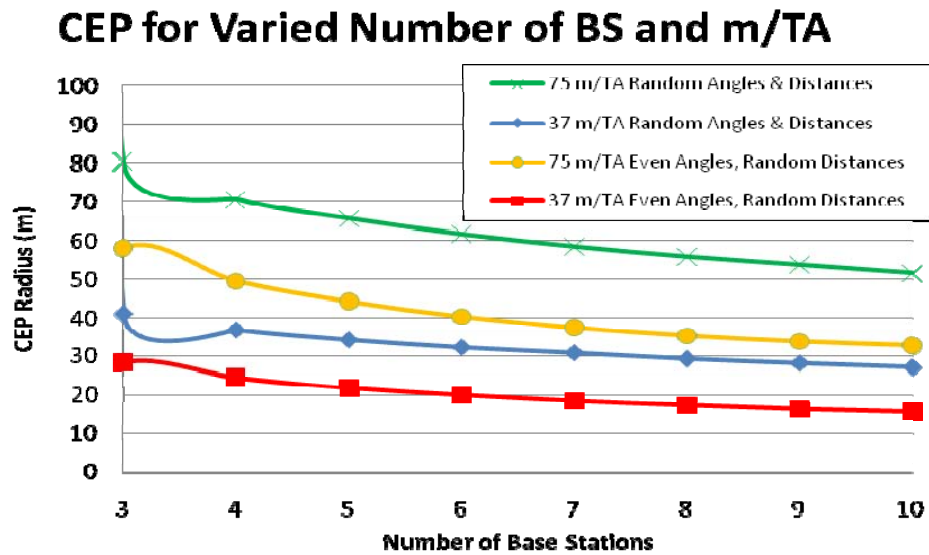


Figure 18. Circular Error Probable from Multiple BS Simulations.

This chapter leveraged the collected real world data from field testing of a WiMAX network and expanded it using computer simulation. Monte Carlo simulation found that using average distance from approximation center point to SS location or CEP as a metric, SS can consistently be located to within the distance per unit TA, less than 50 meters during initial ranging. Noting the TA anomaly in initial range, simulation results showed excellent results for geolocation based on the standards explored in Chapter II, the methods explored in Chapter IV, and the RF collection samples documented in Chapters III and V.

THIS PAGE INTENTIONALLY LEFT BLANK

## **VII. CONCLUSIONS AND RECOMMENDATIONS**

### **A. CONCLUSIONS**

Both laboratory and field testing indicate a high linear correlation to distance with low variance in the TA observed in the initial ranging RNG-RSP message. Based on this result, it is possible to establish a range radius from the BS. Using multiple range radii from several BSs, or other devices capable of ranging the known WiMAX PN codes, crossing radii can establish very accurate locations.

Results of simulation consistently showed that given the high resolution of TA subdivisions within the signed 32-bit value and low variance at fixed distances, geolocation should consistently be practical within a 50-meter CEP dependant on accurate knowledge of TA offsets and values and BS tower locations. At the same time, while high range resolution and low variance are extremely promising, the stipulations of knowledge of TA anomalies and tower locations are non-trivial and important to consider in fielding a geolocation system.

Having accurate BS locations to begin calculations is important, because any error in tower location will be reflected in offsetting all intersections based on that BS. Assuming BS locations to be known and cataloged in a database, it may be possible to add a known initial TA offset from differences in transmit and receive channels, cabling, and other site unique delays, but this too would necessitate site surveys, probably beyond the level of detail typically conducted by a cellular network provider.

Also, while observed initial ranging showed TA radius rings with twice the resolution calculated based on Equation 1 in Chapter II, the round trip delay phenomenon may not appear when extrapolating these results to other periodic ranging occurrences during continued network operation after initial entry. The fundamental methods and principles to extract geolocation from TA data would remain the same, but using the correct distance per unit TA is essential to correct geolocation.



With these practicalities in mind, the results of testing and simulation remain extremely promising. The interoperability standards established by the WiMAX forum provide that, taking due diligence, these results should easily extend to other vendor's equipment and the near verbatim replication of verbiage in the 802.16d-2004 and 802.16e standards suggests easy adaption of this methodology to mobile WiMAX taking into account adjustments to physical layer dependent parameters. Furthermore, WiMAX's chief competitor for 4G cellular, Long Term Evolution (LTE), also operates in a very similar manner, featuring FDD and TDD profiles, of which the TDD internals could similarly be used to geolocate just as explored here for WiMAX networks.

Using the principles explored through testing and simulation, it is possible to geolocate a WiMAX SS within approximately 50 meters, offering potentially 10 times better location accuracy than the GSM methods previously explored in literature, and providing a great capability to be further developed and explored for use both by emergency response personnel and tactical users.

## **B. RECOMMENDATIONS**

Future research should confirm the application of these findings to mobile WiMAX networks and further investigating periodic ranging and other opportunities to extract geolocation data from WiMAX signal internals. Collection and fixes based on actual multi-BS networks, rather than simulation alone, should be conducted to verify and solidify the results in this thesis. Actual collection in real-world multi-tower networks will also motivate the issue of identifying and caching unique identifiers for different towers and subscribers to properly associate traffic to its originator, which would be of critical importance to a fieldable system. Related work may also apply these methods to LTE wireless device, requiring an investigation of the LTE standards before applying the techniques illustrated in this thesis applied to WiMAX.

## APPENDIX I. WIMAX CERTIFICATION PROFILES

Band Index	Frequency Band	Channel Bandwidth	OFDM FFT Size	Duplexing	Notes
<b>Fixed WiMAX Profiles</b>					
1	3.5 GHz	3.5MHz	256	FDD	Products already certified
		3.5MHz	256	TDD	
		7MHz	256	FDD	
		7MHz	256	TDD	
2	5.8GHz	10MHz	256	TDD	
<b>Mobile WiMAX Profiles</b>					
1	2.3GHz–2.4GHz	5MHz	512	TDD	Both bandwidths must be supported by mobile station (MS)
		10MHz	1,024	TDD	
		8.75MHz	1,024	TDD	
2	2.305GHz–2.320GHz, 2.345GHz–2.360GHz	3.5MHz	512	TDD	
		5MHz	512	TDD	
		10MHz	1,024	TDD	
3	2.496GHz–2.69GHz	5MHz	512	TDD	Both bandwidths must be supported by mobile station (MS)
		10MHz	1,024	TDD	
4	3.3GHz–3.4GHz	5MHz	512	TDD	
		7MHz	1,024	TDD	
		10MHz	1,024	TDD	
5	3.4GHz–3.8GHz, 3.4GHz–3.6GHz, 3.6GHz–3.8GHz	5MHz	512	TDD	
		7MHz	1,024	TDD	
		10MHz	1,024	TDD	

Table 4. Fixed and Mobile WiMAX Certification Profiles (From [10]).

THIS PAGE INTENTIONALLY LEFT BLANK

## APPENDIX II. 802.16D-2004 OFDM SYMBOL PARAMETERS

Parameter	Value
$N_{FFT}$	256
$N_{used}$	200
$n$	For channel bandwidths that are a multiple of 1.75 MHz then $n = 8/7$ else for channel bandwidths that are a multiple of 1.5 MHz then $n = 86/75$ else for channel bandwidths that are a multiple of 1.25 MHz then $n = 144/125$ else for channel bandwidths that are a multiple of 2.75 MHz then $n = 316/275$ else for channel bandwidths that are a multiple of 2.0 MHz then $n = 57/50$ else for channel bandwidths not otherwise specified then $n = 8/7$
$G$	1/4, 1/8, 1/16, 1/32
Number of lower frequency guard subcarriers	28
Number of higher frequency guard subcarriers	27
Frequency offset indices of guard subcarriers	-128,-127,...,-101 +101,+102,...,127
Frequency offset indices of pilot carriers	-88,-63,-38,-13,13,38,63,88
Subchannel Index:	Allocated Frequency offset indices of subcarriers:
<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <math>0b10000:</math> </div> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <math>\left\{ \begin{array}{l} 0b01000: \\ 0b11000: \end{array} \right.</math> </div> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <math>\left\{ \begin{array}{l} 0b00100: \\ 0b01100: \end{array} \right.</math> </div> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <math>\left\{ \begin{array}{l} 0b00010: \\ 0b00011: \\ 0b00101: \\ 0b00110: \\ 0b00111: \\ 0b01001: \end{array} \right.</math> </div> <div style="margin-right: 10px;"> <math>\left\{ \begin{array}{l} 0b01010: \\ 0b01011: \\ 0b01101: \\ 0b01110: \\ 0b01111: \\ 0b10001: \end{array} \right.</math> </div> <div style="margin-right: 10px;"> <math>\left\{ \begin{array}{l} 0b10010: \\ 0b10011: \\ 0b10101: \\ 0b10110: \\ 0b10111: \\ 0b11001: \end{array} \right.</math> </div> <div style="margin-right: 10px;"> <math>\left\{ \begin{array}{l} 0b11010: \\ 0b11011: \\ 0b11101: \\ 0b11110: \\ 0b11111: \end{array} \right.</math> </div> </div> </div> </div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <math>\left\{ \begin{array}{l} 0b00001: \\ 0b00011: \\ 0b00101: \\ 0b00110: \\ 0b00111: \\ 0b01001: \end{array} \right.</math> </div> <div style="margin-right: 10px;"> <math>\left\{ \begin{array}{l} 0b01010: \\ 0b01011: \\ 0b01101: \\ 0b01110: \\ 0b01111: \\ 0b10001: \end{array} \right.</math> </div> <div style="margin-right: 10px;"> <math>\left\{ \begin{array}{l} 0b10010: \\ 0b10011: \\ 0b10101: \\ 0b10110: \\ 0b10111: \\ 0b11001: \end{array} \right.</math> </div> <div style="margin-right: 10px;"> <math>\left\{ \begin{array}{l} 0b11010: \\ 0b11011: \\ 0b11101: \\ 0b11110: \\ 0b11111: \end{array} \right.</math> </div> </div> <p>Note that pilot subcarriers are allocated only if two or more subchannels are allocated.</p>

Table 5. OFDM Symbol Parameters (From [12]).

THIS PAGE INTENTIONALLY LEFT BLANK

## APPENDIX III. GUI HTML/JAVASCRIPT CODE

```
<html>
<!-- LT D. E. Barber, NPS GSEAS -->
<!-- WiMAX Geolocation Web Interface -->
<!-- Current Revision 9.11b, 11 November 2009 -->

<head>
  <title> OWL (v9.11b) </title>

  <script
src="http://maps.google.com/maps?file=api&v=2&sensor=false&key=ABQIAAAAKg2XhyFKbQwX6KYN9UftGBRzqH3tuoONQrJc0Yfxh_5EF-
rAQBSuBt6V6Vzrd3cmnuVAjYdEHR3ImQ"
type="text/javascript"></script>

</head>

<!-- Displayed Header -->

<body style="background-color:D3D3D3" onload="GUnload()">
  <noscript>Browser does not have JavaScript enabled!</noscript>
  
  
  <h1 style="text-align:center"><i>O</i>FDM <i>W</i>ireless
  <i>L</i>ocator </h1>
  <p align="center"> <i>Version 9.11b</i><br/>
  TA is set for <i>BW</i>= 3.5 MHz. Please enter latitude/longitude
  in          decimal          degrees          (<a
href="http://www.fcc.gov/mb/audio/bickel/DDDMSS-decimal.html/"
target="_blank">converter</a>).</p>

<!-- Table of Fields to Recieve Input -->

<form name="inputForm">
<table align="center" summary="Data entry fields."> <tr>

<td><b>SITE 1 (Base Station)</b></td>
  <td>Lat:</td><td> <input type="text" name="S1LAT"/></td>
  <td>Lng:</td><td> <input type="text" name="S1LNG"/></td>
  <td></td>
  <td>Offset:</td><td> <input type="text" maxlength="10" name="S1OFF"/>
</td>
  <td>TA:</td><td> <input type="text" maxlength="10" name="S1TA"/></td>
</tr><tr>

<td><b>SITE 2 (Collector)</b></td>
  <td>Lat:</td><td> <input type="text" name="S2LAT"/></td>
  <td>Lng:</td><td> <input type="text" name="S2LNG"/></td>
  <td></td> <td></td> <td></td>
  <td>TA:</td><td><input type="text" maxlength="10" name="S2TA"/></td>
```



```

// Number of Points to Use in Circle Approximation
npts = 40;
// Distribution Weighting for Probability Ellipse
eProbDist = 1.2;

// Site Locations from Form
s1lat = inputForm.S1LAT.value;
s1lng = inputForm.S1LNG.value;
s2lat = inputForm.S2LAT.value;
s2lng = inputForm.S2LNG.value;

/*****
* Range Radius Calculations *
*****/
// Calculate Circles Radii
// Site 1
slta = inputForm.S1TA.value - inputForm.S1OFF.value;
slr = slta * mTA; // Site 1 Range Radius in meters
slrLat = slr / mLat; // Latitude Variance from Radius
slrLng = slr / mLng; // Longitude Variance from Radius

//Site 2
s2r = inputForm.S2TA.value * mTA;
s2rLat = s2r / mLat; // Lat/Lng Delta from Radius
s2rLng = s2r / mLng;

// Invalid Timing Adjust Input Warning
if(slta<0||inputForm.S2TA.value<0){
    alert("Input Error: Net Negative Timing Adjust");}

// Create Circle Arrays
var slcLat=new Array();
var slcLng=new Array();
var s2cLat=new Array();
var s2cLng=new Array();

// Create Arrays to Hold Gmap Lat/Lng
var s1Glatlng=new Array();
var s2Glatlng=new Array();

// Loop to Populate Radii Lat/Lng Arrays
for(var i=0; i<= npts; i++)
{
    angle=i*2*pi/npts; //Angle in Radians
    cosangle = Math.cos(angle); //Math Function for Cosines
    sinangle = Math.sin(angle);

    // Calculate Circles in Rectangular Coordinates
    /* x1 Forces Addition vice String Concatenation */
    slcLat[i]=s1lat *1 + (slrLat * cosangle);
    slcLng[i]=s1lng *1 + (slrLng * sinangle);
    s2cLat[i]=s2lat *1 + (s2rLat * cosangle);
    s2cLng[i]=s2lng *1 + (s2rLng * sinangle);
}

```



```

    // Create Gmaps Coordinate Arrays
    s1GlatLng[i]=new GLatLng(s1cLat[i],s1cLng[i]);
    s2GlatLng[i]=new GLatLng(s2cLat[i],s2cLng[i]);
}

/*****
* High Probability Polygon Approximation *
*****/

// Inter Site Distance via Pythag
deltaLat=(s2lat-s1lat)*mLat;
deltaLng=(s2lng-s1lng)*mLng;
dist=Math.sqrt(deltaLat*deltaLat+deltaLng*deltaLng);

// Angle Set to 0 = North
phi=Math.atan2(deltaLng,deltaLat);
cosphi=Math.cos(phi);
sinphi=Math.sin(phi);
bearS1toS2=Math.round(phi*180/pi); //Displayed Output Only
if (bearS1toS2<0){
    bearS1toS2=bearS1toS2+360;}

// Calculate Intersection
if(dist>s1r+s2r){
    methodAlert="Seperated Radii";
    deltaR=dist-s1r-s2r;
    mpLat=s1lat*1+cosphi*(s1r+0.5*deltaR)/mLat;
    mpLng=s1lng*1+sinphi*(s1r+0.5*deltaR)/mLng;
    major=deltaR*eProbDist;
    minor=major;
    psi=0;
}

else if(dist<Math.abs(s1r-s2r)){
    methodAlert="Contained Circles";
    if(s1r>s2r){
        deltaR=s1r-s2r-dist;
        mpLat=s1lat*1+cosphi*(s1r-0.5*deltaR)/mLat;
        mpLng=s1lng*1+sinphi*(s1r-0.5*deltaR)/mLng;
    }
    else{
        deltaR=s2r-s1r-dist;
        mpLat=s1lat*1+cosphi*(-s1r-0.5*deltaR)/mLat;
        mpLng=s1lng*1+sinphi*(-s1r-0.5*deltaR)/mLng;
    }
    major=deltaR*eProbDist;
    minor=major;
    psi=0;
}

else{
    methodAlert="Intercepts";
    // Distance from Site 1 to Midpoint
    slmp=(s1r*s1r-s2r*s2r+dist*dist)/(2*dist);
    // Distance from Midpoint to Intersection

```

```

mp2i=Math.sqrt(slr*slr-slmp*slmp);

// Midpoint Coordinates
mpLat=s1lat*1+slmp*(s2lat-s1lat)/dist;
mpLng=s1lng*1+slmp*(s2lng-s1lng)/dist;

// Intersection Coordinates
/* Note Final Coordinate Transform Constants */
int1Lat=mpLat*1+mp2i*(s2lng-s1lng)/dist*(mLng/mLat);
int1Lng=mpLng*1-mp2i*(s2lat-s1lat)/dist*(mLat/mLng);
int2Lat=mpLat*1-mp2i*(s2lng-s1lng)/dist*(mLng/mLat);
int2Lng=mpLng*1+mp2i*(s2lat-s1lat)/dist*(mLat/mLng);

// Angle Between Intercepts is Pi/2 from Intersite Angle
psi=phi+(pi/2);

// Define Ellipse Axis Lengths
// Adjust Midpoint if Both Foci within One Radius
if (dist>slr&&dist>s2r){
minor=(slr+s2r-dist)*eProbDist;
}
else if (s2r<slr){
minor=(dist+s2r-slr)*eProbDist;
mpLat=s1lat*1+cosphi*slr/mLat;
mpLng=s1lng*1+sinphi*slr/mLng;
}
else {
minor=(dist+slr-s2r)*eProbDist;
mpLat=s2lat*1-cosphi*s2r/mLat;
mpLng=s2lng*1-sinphi*s2r/mLng;
}

major=(minor+mp2i)*eProbDist;
}

// Calculate Probability Ellipse Bounds

// Create Ellipse Arrays
var peLat=new Array();
var peLng=new Array();
var peGlatlng=new Array();

cospsi=Math.cos(psi); // Angle Parameters of Ellipse
sinpsi=Math.sin(psi); // Constant So Outside Loop

halfnpts=npts/2; // Use 1/2 Number Points in Circle for Ellipse

for(k=0; k<=halfnpts; k++){
    beta=k*4*pi/npts; //Parametric Angle in Radians
    cosbeta=Math.cos(beta);
    sinbeta=Math.sin(beta);

    // Ellipse Defined in Rectangular Coordinates
    peLat[k]=mpLat*1 + (major*cosbeta*cospsi -

```

```

        minor*sinbeta*sinpsi)/mLat;
    peLng[k]=mpLng*1+ (major*cosbeta*sinpsi +
        minor*sinbeta*cospsi)/mLng;

    // Create Gmaps Coordinate Array
    peGlatlng[k]=new GLatLng(peLat[k],peLng[k]);
}

// Distance from Ellipse Center to Marker via Pythag
cmLat=(mpLat-inputForm.MLAT.value)*mLat;
cmLng=(mpLng-inputForm.MLNG.value)*mLng;
cmdist=Math.sqrt(cmLat*cmLat+cmLng*cmLng);

/*****
* Output and Display *
*****/

// Map Initialization Function
if (GBrowserIsCompatible()) {
    var map = new GMap2(document.getElementById("map_canvas"));
    map.setMapType(G_PHYSICAL_MAP);
    map.setCenter(new GLatLng(inputForm.S1LAT.value,
        inputForm.S1LNG.value), 14);
    map.setUIToDefault();

// Site Radius Circles
    var s1circle = new GPolyline(s1Glatlng,"#ff00ff",5);
    map.addOverlay(s1circle);

    var s2circle = new GPolyline(s2Glatlng,"#0000ff",5);
    map.addOverlay(s2circle);

// Probability Ellipse
    var probell = new GPolygon(peGlatlng,"#f33f00",5,1,"ff0000",0.33);
    map.addOverlay(probell);

// Marker for Subscriber Location if Known
/* Null Input Defaults to Equator at Prime Meridan */
    var mark = new GLatLng(inputForm.MLAT.value,
        inputForm.MLNG.value);
    var iconSS = new GIcon();
    iconSS.image = "images/redSS.png";
    iconSS.shadow = "images/shadow.png";
    iconSS.iconSize = new GSize(32, 37);
    iconSS.shadowSize = new GSize(51, 37);
    iconSS.iconAnchor = new GPoint(16, 37);
    SStitle = "Subscriber Station \rDist to CP: " +
        Math.round(cmdist) + " m";
    map.addOverlay(new GMarker(mark,{icon:iconSS,
        title:SStitle}));

// Rounded Site Radii for Display
    slrR=Math.round(slr);
    s2rR=Math.round(s2r);

```

```

// Markers for Site Locations
var sitel = new GLatLng(s1lat,s1lng);
var iconBSpink = new GIcon();
    iconBSpink.image = "images/pinkBS.png";
    iconBSpink.shadow = "images/BSshadow.png";
    iconBSpink.iconSize = new GSize(22, 32);
    iconBSpink.shadowSize = new GSize(39, 32);
    iconBSpink.iconAnchor = new GPoint(11, 16);
    BS1title = "Base Station (S1) \rRadius: " + s1rR +
        " m\rS2 Bearing: " + bearS1toS2;
map.addOverlay(new GMarker(sitel,{icon:iconBSpink,
    title:BS1title}));

var site2 = new GLatLng(s2lat,s2lng);
var iconBSblue = new GIcon();
    iconBSblue.image = "images/blueBS.png";
    iconBSblue.shadow = "images/BSshadow.png";
    iconBSblue.iconSize = new GSize(22, 32);
    iconBSblue.shadowSize = new GSize(39, 32);
    iconBSblue.iconAnchor = new GPoint(11,16);
    BS2title = "Site 2\rRadius: " + s2rR + " m";
map.addOverlay(new GMarker(site2,{icon:iconBSblue,
    title:BS2title}));

// Detail Alert Message
if (inputForm.alertCheck.checked){
    // Straight Line Between Sites
    var centerLine = new GPolyline([
        new GLatLng(s1lat,s1lng),
        new GLatLng(s2lat,s2lng)
    ],"#ffff00",5);
    map.addOverlay(centerLine);

    mpLatR=Math.round(mpLat*10000)/10000;
    mpLngR=Math.round(mpLng*10000)/10000;

    // Mark Center Point of Approximation Ellipse
    var MPmark = new GLatLng(mpLat,mpLng);
    CPtitle = "Ellipse Center Point\rLat: "
        + mpLatR + "\rLng: " + mpLngR;
    map.addOverlay(new GMarker(MPmark,{title:CPtitle}));

var endTime = new Date();

alert(
    "Time to Execute Script: "
    + (endTime.getTime() - startTime.getTime()) + " ms " +
    "\rPoints Utilized in Radius Circles: " + npts + " " +
    "\rCorrected Site 1 TA: " + slta + " 1/Fs " +
    "\rMeters per Degree Longitude: " + Math.round(mLng) + " " +
    "\r\rDistance Between Sites: " + Math.round(dist) + " m " +
    "\rBearing from Site 1 to Site 2: " + bearS1toS2 + " " +
    "\rSite 1 Radius: " + slrR + " m " +

```

```

        "\rSite 2 Radius:  " + s2rR + " m " +
        "\r\rMethod Used: " + methodAlert + " " +
        "\r\rEllipse Midpoint Lat: " + mpLatR + " " +
        "\rEllipse Midpoint Lng: " + mpLngR + " " +
        "\r\rDistance from Midpoint to Marker: "
        + Math.round(cmdist) + " m"
    );
    }
}
}
// -->
</script>

<!-- Draw Map -->
    <div id="map_canvas" style="center; height: 512px"></div>

</body>
</html>

```

## APPENDIX IV. FIELD TEST IMAGES



Figure 19. Base Station.



Figure 20. Subscriber Station.



Figure 21. Collection Vehicle with Roof Mounted Antenna.



Figure 22. Collection Suite (GPS, Laptop, and WaveJudge).

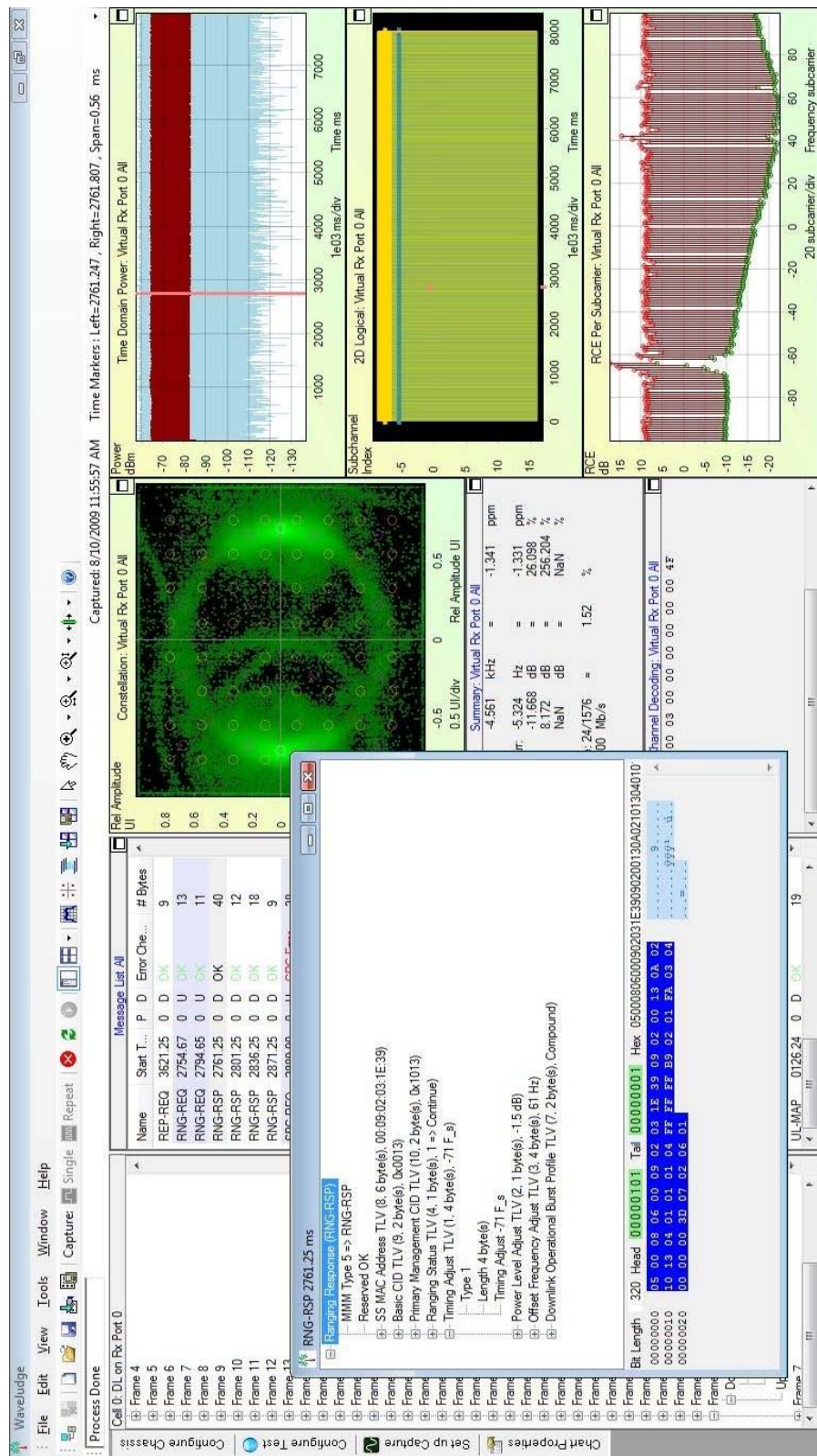


Figure 23. Screen Shot of WaveJudge Interface with Captured RNG-RSP Open.



THIS PAGE INTENTIONALLY LEFT BLANK

## APPENDIX V. MULTIPLE BASE STATION SIMULATIONS

### A. TWO BASE STATIONS THROUGH VARYING ANGLES

#### 1. Two Base Stations Through Varying Angle MATLAB Code

```
% Two Base Station Simulation Varying Angle
% LT D. E. Barber, NPS GSEAS
% Rev. 12, 1 Sept 2009
clear all;

% Measured Timing Adjust Parameters Based on Field Measurements
mTA = 37;           %Mean meters per Unit of Timing Adjust
stdTA = 0.673;      %Mean TA standard deviation

% Synthetic Base Station Distance Parameters
SiteMean = 1000;    %Mean Site Distance
SiteSD = 300;       %Site Standard Deviation

circlePts = 30;     % Number of points in circle plots
t = (0:circlePts)*2*pi/circlePts;

% Initialize Arrays
AngAvg = 0;         errorDist = 0;

for i = 0:18         %Iterate Angles in 10 Degree Steps
    theta = i*pi/18; %Angle from Subscriber to Base Station 2

    for j = 1:100000 %Repeated Runs to Average

        % Distance to Simulated Base Stations with X/Y Coordinates
        DistBS1 = SiteSD*randn(1) + SiteMean;    x1=DistBS1;          y1=0;
        %BS on Axis
        DistBS2 = SiteSD*randn(1) + SiteMean;    x2=cos(theta)*DistBS2;
        y2=sin(theta)*DistBS2;

        % Timing Adjust for Base Stations including Random Variation
        TABS1 = round(DistBS1/mTA) + round(stdTA*randn(1));
        TABS2 = round(DistBS2/mTA) + round(stdTA*randn(1));

        % Site Radii based on TA
        slr = TABS1 * mTA;  s2r = TABS2 * mTA;

        % Inter-Site Distance via Pythag and Angle from ATAN2
        dist = sqrt((x2-x1)^2 + (y2-y1)^2);
        phi = atan2((y2-y1),(x2-x1));

        % Find Midpoint Between Circles or Circle Intersections
        if(dist>slr+s2r) % Separated Radii
```

```

    deltaR=dist-s1r-s2r;
    mpX=x1+cos(phi)*(s1r+0.5*deltaR);
    mpY=y1+sin(phi)*(s1r+0.5*deltaR);
elseif (dist<abs(s1r-s2r)) % Contained Circles
    if(s1r>s2r)
        deltaR=s1r-s2r-dist;
        mpX=x1+cos(phi)*(s1r-0.5*deltaR);
        mpY=y1+sin(phi)*(s1r-0.5*deltaR);
    else
        deltaR=s2r-s1r-dist;
        mpX=x1+cos(phi)*(-s1r-0.5*deltaR);
        mpY=y1+sin(phi)*(-s1r-0.5*deltaR);
    end
else % Intersections
    slmp=(s1r*s1r-s2r*s2r+dist*dist)/(2*dist); % Distance BS1 to Midpoint
    mpX=x1+slmp*(x2-x1)/dist;
    mpY=y1+slmp*(y2-y1)/dist;
end

% Distance from Midpoint to Subscriber at Origin
errorDist(j) = sqrt(mpX^2+mpY^2);

end

AngAvg(i+1)=mean(errorDist);

% Plot Once Every Other Angle
if mod(i,2)==0
figure(i/2+1);
clf;
hold on;
axis equal;
title(['Base Station 1 on X Axis, Base Station 2 at ',...
int2str(i*10), ' Degrees']);
xlabel('Meters');ylabel('Meters');
plot(s1r*cos(t)+ x1, s1r*sin(t)+ y1,'-b');
plot(s2r*cos(t)+ x2, s2r*sin(t)+ y2,'-m');
plot(x1,y1,'^b','MarkerSize',10); plot(x2,y2,'^m','MarkerSize',10);
plot(mpX,mpY,'rp','MarkerSize',10); plot(0,0,'sk','MarkerSize',10);
hold off;
end

end

```

## 2. Two Base Stations through Varying Angles Example Plots

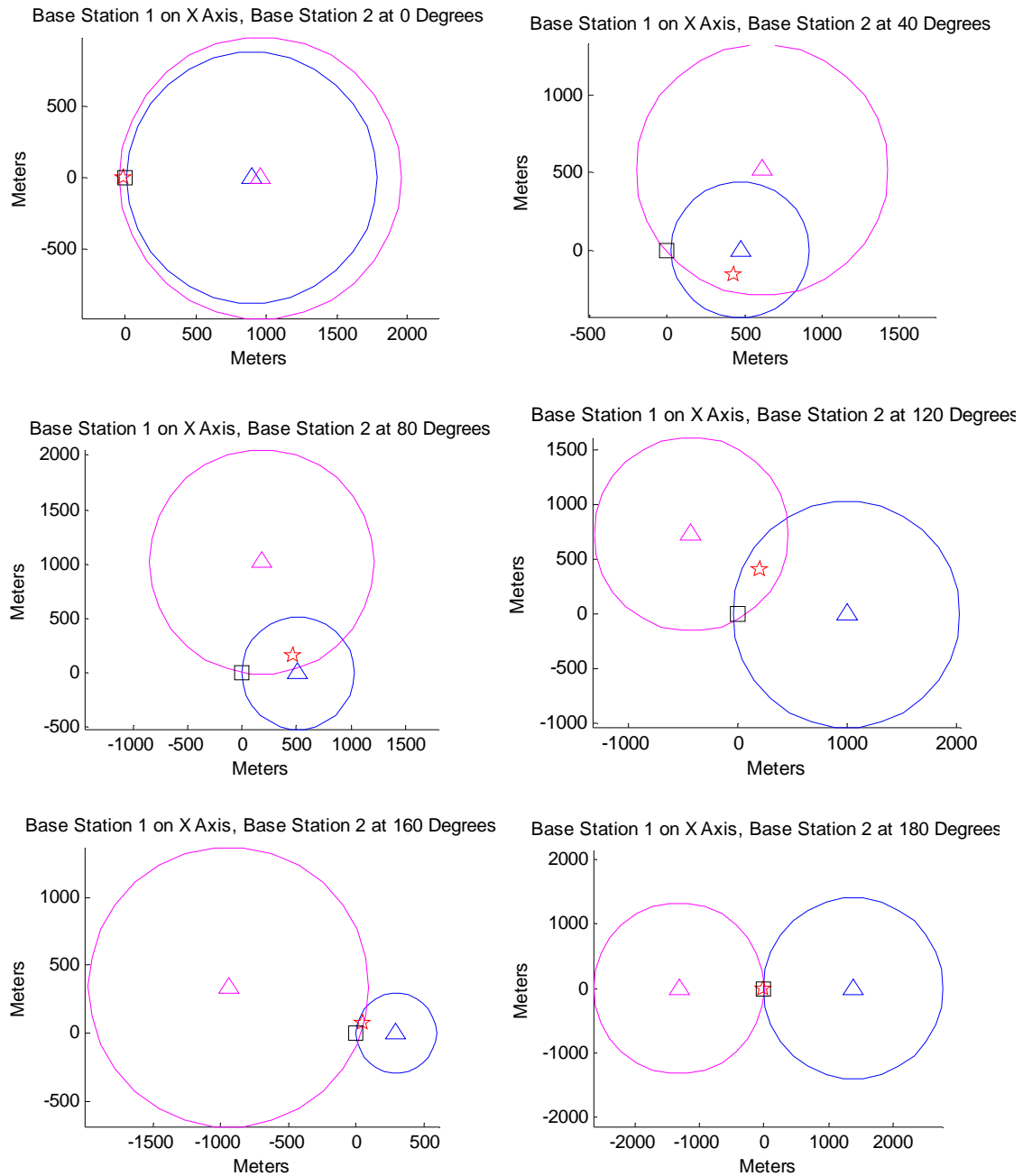


Figure 24. Sample Plots from 2 BS Simulation with Varying Angles and Distances.

(Triangles denote the BSs, a square indicates the SS location , and the star shows approximation center point.)

## B. MULTIPLE BASE STATIONS

### 1. Multiple Base Stations MATLAB Code

```
% Multiple Base Station Simulation
% LT D. E. Barber, NPS GSEAS
% Rev. 14, 11 Nov 2009
clear all;

% Simulation Parameters
MaxNumberBS = 10; % Maximum Number of Base Stations to Simulate
Runs = 100000;    % Number of Runs at Each Number of Base Stations

% Measured Timing Adjust Parameters Based on Field Measurements
mTA = 37;        % Mean meters per Unit of Timing Adjust
stdTA = 0.673;   % Mean TA standard deviation

% Synthetic Base Station Distance Parameters
SiteMean = 1200; % Mean Site Distance
SiteSD = 400;    % Site Standard Deviation

circlePts = 180; % Number of points in circle plots
t = (0:circlePts)*2*pi/circlePts;

AvgError = 0:MaxNumberBS-2; % Average Error at N Base Stations
StdError = 0:MaxNumberBS-2; % Standard Deviation at N Base Stations
CEP = 0:MaxNumberBS-2;      % CEP at N Base Stations

% Number of Base Stations to Simulate
for N = 2:MaxNumberBS

% Initialize Sized Arrays
DistBS = zeros(1,N); % Array to Hold Distance to Base Stations
bsX = zeros(1,N);    % Array of Base Station X Coordinates
bsY = zeros(1,N);    % Array of Base Station Y Coordinates
bsTA = zeros(1,N);   % Array of Timing Adjusts
sr = zeros(1,N);     % Array of Site Radius
int1X = zeros(1,N);  % Array for X Coordinate of First Intercept
int1Y = zeros(1,N);  % Array for Y Coordinate of First Intercept
int2X = zeros(1,N);  % Array for X Coordinate of Second Intercept
int2Y = zeros(1,N);  % Array for Y Coordinate of Second Intercept
intX = zeros(1,N);   % Array for Chosen Intercept from Pair's X Coord
intY = zeros(1,N);   % Array for Chosen Intercept from Pair's Y Coord
DSE = zeros(1,Runs); % Array of Distances to Estimate Center Point

for h = 1:Runs

% Create Base Stations
for i = 1:N % Number of Base Stations
DistBS(i) = SiteSD*randn(1) + SiteMean; % Base Station Distance
theta = 2*pi*rand(1); % Angle from Subscriber
bsX(i) = cos(theta)*DistBS(i); % X Coordinate
```

```

bsY(i) = sin(theta)*DistBS(i); % Y Coordinate
bsTA(i) = round(DistBS(i)/mTA) + round(stdTA*randn(1)); % Timing Adjust
sr(i) = bsTA(i) * mTA; % Radius from Timing Adjust
end

% Find Two Intersections for Each Pair of Radii
for a = 1:N
    b = mod(a,N)+1;

    % Inter-Site Distance via Pythag and Angle from ATAN2
    dist = sqrt((bsX(b)-bsX(a))^2 + (bsY(b)-bsY(a))^2);
    phi = atan2((bsY(b)-bsY(a)),(bsX(b)-bsX(a)));

    % Find Midpoint Between Circles or Circle Intersections
    if(dist>sr(a)+sr(b)) % Separated Radii
        deltaR=dist-sr(a)-sr(b); % Closest Point Set as Both Intersections
        int1X(a)=bsX(a)+cos(phi)*(sr(a)+0.5*deltaR);
        int1Y(a)=bsY(a)+sin(phi)*(sr(a)+0.5*deltaR);
        int2X(a)=int1X(a); int2Y(a)=int1Y(a);
    elseif (dist<abs(sr(a)-sr(b))) % Contained Circles
        if(sr(a)>sr(b)) % Narrowest Set as Both Intersections
            deltaR=sr(a)-sr(b)-dist;
            int1X(a)=bsX(a)+cos(phi)*(sr(a)-0.5*deltaR);
            int1Y(a)=bsY(a)+sin(phi)*(sr(a)-0.5*deltaR);
            int2X(a)=int1X(a); int2Y(a)=int1Y(a);
        else
            deltaR=sr(b)-sr(a)-dist;
            int1X(a)=bsX(a)+cos(phi)*(-sr(a)-0.5*deltaR);
            int1Y(a)=bsY(a)+sin(phi)*(-sr(a)-0.5*deltaR);
            int2X(a)=int1X(a); int2Y(a)=int1Y(a);
        end
    else % Intersections
        slmp=(sr(a)^2-sr(b)^2+dist^2)/(2*dist); % Dist from BS1 to Midpoint
        mp2i=sqrt(sr(a)^2-slmp^2); % Dist Midpoint to Intercept
        mpX=bsX(a)+slmp*(bsX(b)-bsX(a))/dist; % Midpoint X Coordinate
        mpY=bsY(a)+slmp*(bsY(b)-bsY(a))/dist; % Midpoint Y Coordinate
        int1X(a)=mpX+mp2i*(bsY(b)-bsY(a))/dist; % First Intersection Coord
        int1Y(a)=mpY-mp2i*(bsX(b)-bsX(a))/dist;
        int2X(a)=mpX-mp2i*(bsY(b)-bsY(a))/dist; % Second Intersection Coord
        int2Y(a)=mpY+mp2i*(bsX(b)-bsX(a))/dist;
    end
end

% Select Closest Proximity Intersection from Each Pair
for j = 1:N
    k = mod(j+1,N)+1;

    distFirst=sqrt((int1X(k)-int1X(j))^2+(int1Y(k)-int1Y(j))^2);
    dist12=sqrt((int2X(k)-int1X(j))^2+(int2Y(k)-int1Y(j))^2);
    if (dist12 < distFirst)
        distFirst = dist12;
    end

    distSecond=sqrt((int1X(k)-int2X(j))^2+(int1Y(k)-int2Y(j))^2);

```

```

dist22=sqrt((int2X(k)-int2X(j))^2+(int2Y(k)-int2Y(j))^2);
if (dist22 < distSecond)
    distSecond = dist22;
end

if (distFirst < distSecond)    % Store Selected Intersection in Array
    intX(j) = int1X(j); intY(j) = int1Y(j);
else
    intX(j) = int2X(j); intY(j) = int2Y(j);
end
end

% Approx Center of Polygon from Selected Intersections by Coord Mean
EstimateX = mean(intX);
EstimateY = mean(intY);

%Distance from Subscriber to Estimate
DSE(h) = sqrt(EstimateX^2+EstimateY^2);
end

AvgError(N-1) = mean(DSE);      % Mean Estimate Error at N Stations
StdError(N-1) = std(DSE);      % Standard Deviation of Error
CEP(N-1) = median(DSE);        % Circular Error Prob at N Stations

% Plot
figure(N-1)
clf;
hold on;
axis equal;
title([int2str(N), ' Base Stations, ' int2str(DSE(h)), ...
    ' Meters from Estimate to Subscriber']);
xlabel('Meters');ylabel('Meters');
for p = 1:N
    plot(bsX(p),bsY(p),'^b','MarkerSize',8);          % Plot Base Station
    plot(sr(p)*cos(t)+ bsX(p), sr(p)*sin(t)+ bsY(p));% Plot TA Radius
    plot(int1X(p),int1Y(p),'ob','MarkerSize',4);      % Plot Both Int
    plot(int2X(p),int2Y(p),'ob','MarkerSize',4);
    plot(intX(p),intY(p),'rd','MarkerSize',10);       % Mark Selected Int
end
plot(0,0,'sk','MarkerSize',12);                      % Plot Subscriber
plot(EstimateX,EstimateY,'rp','MarkerSize',12);      % Plot Estimated
hold off;

end

```

## 2. Random Angle and Distance Example Plots

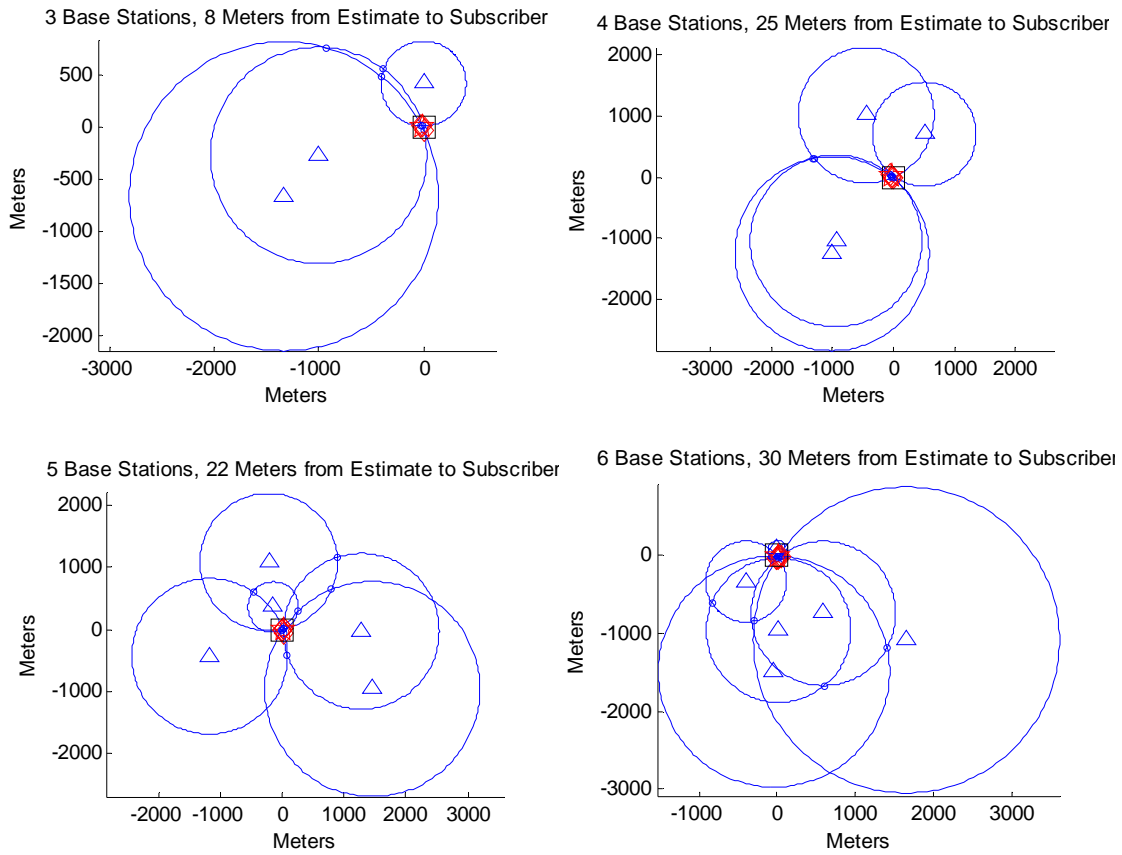


Figure 25. Sample Multiple BS Plots with Random Angles and Distances.

Triangles denote BSs, the square indicates the SS location, dots mark iterated intersections, diamonds mark selected intersections, and a star shows approximation center point.



### 3. Evenly Spaced Angles with Random Distance Example Plots

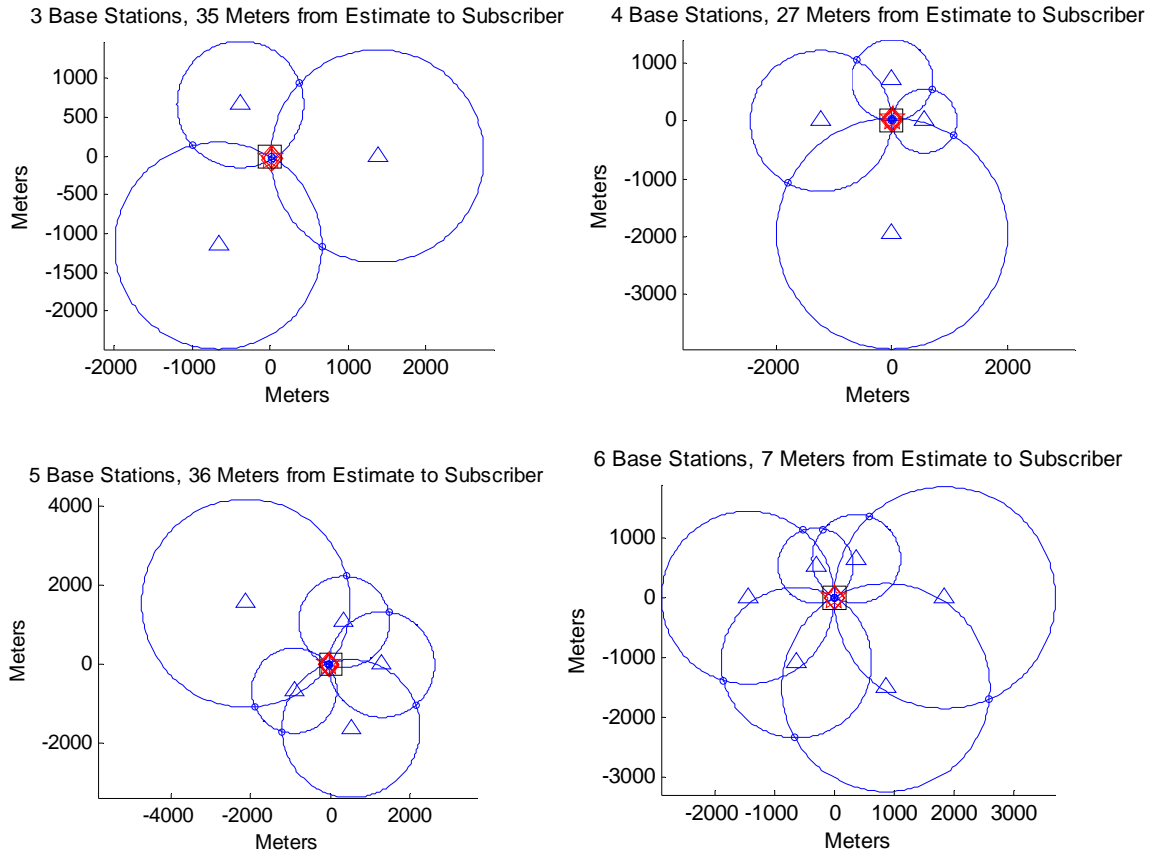


Figure 26. Sample Multiple BS Plots with Equal Angles and Random Distances.

Triangles denote BSs, the square indicates the SS location, dots mark iterated intersections, diamonds mark selected intersections, and a star shows approximation center point.

#### 4. Evenly Spaced Angles with Fixed Radial Distance Example Plots

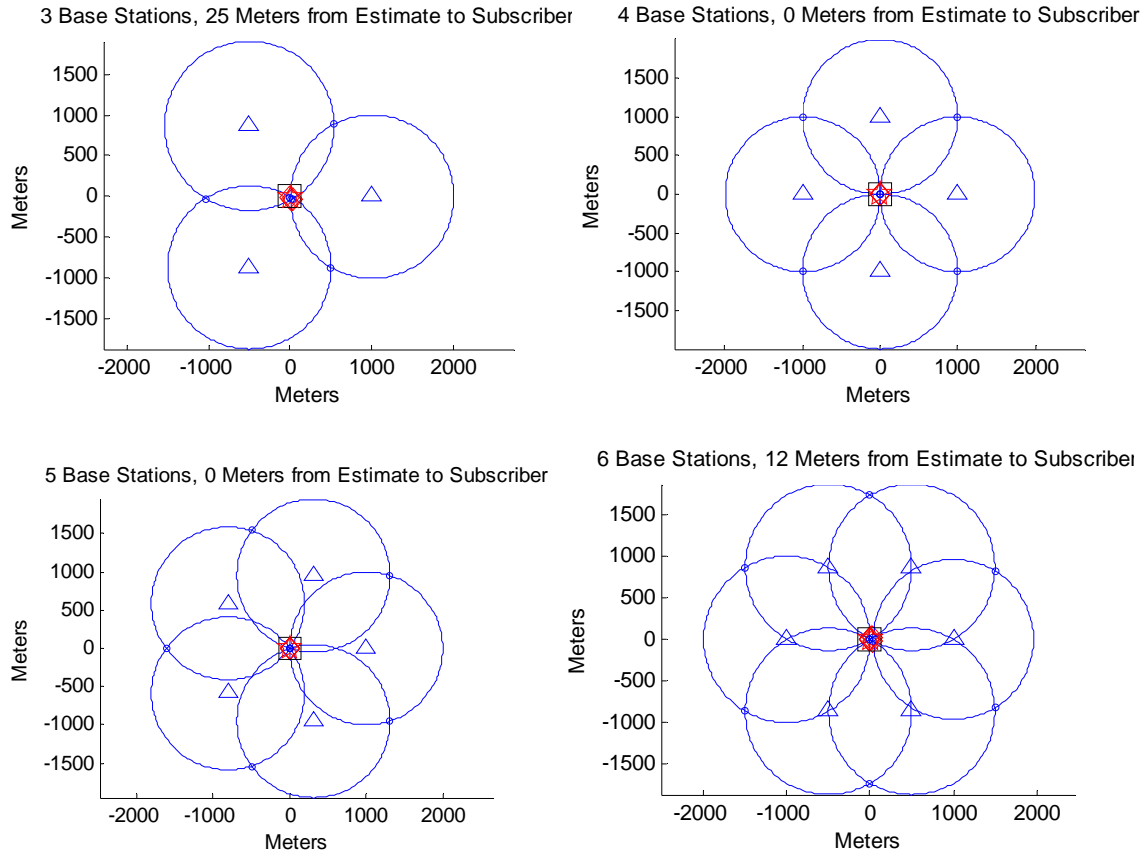


Figure 27. Sample Multiple BS Plots with Equal Angles and Fixed 1 km Distances.

Triangles denote BSs, the square indicates the SS location, dots mark iterated intersections, diamonds mark selected intersections, and a star shows approximation center point.

THIS PAGE INTENTIONALLY LEFT BLANK

## LIST OF REFERENCES

- [1] C. Eklund, R. B. Marks, K. L. Stanwood and S. Wang, "IEEE Standard 802.16: A Technical Overview of the WirelessMAN Air Interface for Broadband Wireless Access," *IEEE Communications Magazine*, pp. 98–107, June 2002.
- [2] B. Bialek, "Analysis of 802.16 WiMAX networks," Naval Postgraduate School, Monterey, California, December 2007.
- [3] WiMAX Forum, "Mobile WiMAX Plugfest," Bechtel Telecommunications, Frederick, Maryland, September 2006.
- [4] Redline Communications. (2005, 11/9). Redline achieves TDD interoperability with four WiMAX vendors at WiMAX Forum Plugfest in Beijing. [Online]. 2009(5/12), Available: [http://www.redlinecommunications.com/news/pressreleases/2005/110905\\_02.html](http://www.redlinecommunications.com/news/pressreleases/2005/110905_02.html)
- [5] WiMAX Forum. (2009, November). WiMAX maps. [Online]. 2009(11/11), Available: <http://www.wimaxmaps.org/>
- [6] Federal Communications Commission, "FCC amended report to congress on the deployment of E-911 Phase II services Tier III Service Providers," April 2005.
- [7] H. H. Loomis, "Geolocation of electromagnetic emitters," Naval Postgraduate School, Monterey, California, November 1999.
- [8] M. A. Spirito and A. G. Mattioli, "Preliminary experimental results of a GSM mobile phones positioning system based on timing advance," in *Vehicular Technology Conference*, 1999, pp. 2072–2076.
- [9] G. P. Yost and S. Panchapakesan, "Improvement in estimation of time of arrival (TOA) from timing advance (TA)," in *IEEE 1998 International Conference on Universal Personal Communications*, 1998, pp. 1367–1372.
- [10] J. G. Andrews, A. Ghosh and R. Muhamed, *Fundamentals of WiMAX: Understanding Broadband Wireless Networking*. Upper Saddle River, New Jersey: Prentice Hall, 2007, pp. 449.
- [11] Institute of Electrical and Electronics Engineers, "IEEE Standard for Local and Metropolitan Area Networks Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems," February 2006.

- [12] Institute of Electrical and Electronics Engineers, "IEEE Standard for Local and Metropolitan Area Networks, Part 16: Air Interface for Fixed Broadband Wireless Access Systems," October 2004.
- [13] X. Leleu, private communication, 1 July 2009.
- [14] Redline Communications. (2008, August). RedMAX base station (AN-100U) system specifications. Redline Communications, Ontario, Canada. [Online]. 2009(7/24), Available:  
[http://www.redlinecommunications.com/news/resourcecenter/productinfo/RedMAX\\_AN100u\\_ds.pdf](http://www.redlinecommunications.com/news/resourcecenter/productinfo/RedMAX_AN100u_ds.pdf)

## **INITIAL DISTRIBUTION LIST**

1. Defense Technical Information Center  
Ft. Belvoir, Virginia
2. Dudley Knox Library  
Naval Postgraduate School  
Monterey, California
3. Chair, Department of Electrical and Computer Engineering  
Naval Postgraduate School  
Monterey, California
4. Professor John McEachen  
Naval Postgraduate School  
Monterey, California
5. Professor Herschel Loomis  
Naval Postgraduate School  
Monterey, California
6. Professor Vicente Garcia  
Naval Postgraduate School  
Monterey, California
7. Paul Greenfield  
National Reconnaissance Office/Advanced Systems & Technology  
Chantilly, Virginia
8. LTJG Tom Jones  
National Reconnaissance Office/Advanced Systems & Technology  
Chantilly, Virginia